**Example of Forced Vibrations**

Here we have the equation for a vibrating string with \( a = 1 \), length is 1, and we are forcing with force 1 and frequency 1. That is we have the system

\[
y_{tt} = y_{xx} + \cos t \\
y(0, t) = y(1, t) = 0 \\
y(x, 0) = y_t(x, 0) = 0
\]

We got a particular solution \( (\cos x - \frac{\cos 1 - 1}{\sin 1} \sin x - 1) \cos t \) and we took the odd 2 periodic extension of the function \( \cos x - \frac{\cos 1 - 1}{\sin 1} \sin x - 1 \) (this was defined on \( 0 < x < 1 \)) and we call the extension \( F \). Then the solution was

\[
y(x, t) = \frac{F(x + t) + F(x - t)}{2} + (\cos x - \frac{\cos 1 - 1}{\sin 1} \sin x - 1) \cos t
\]

Which has the following plot.