MATH 241, Calculus III
Midterm I
19 February 2015

Name: ____________________________________________

- No hats or dark sunglasses. All hats are to be removed.

- All book bags are to be closed and placed in a way that makes them inaccessible. Do not reach into your bag for anything during the exam. If you need extra pencils, pull them out now.

- No cell phones. Turn them off now. If you are seen with a cell phone in hand during the exam, it will be construed as cheating and you will be asked to leave. This includes using it as a time-piece.

- No music systems (IPODs, MP3 players, etc.), calculators, or cell phones.

- The exam is worth a total of 100 points. Check to see that you have all nine problems.

- **Show ALL your work and reasoning** to receive full credit.

- Good luck. You have **50 minutes** to complete the exam.

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<th>Question</th>
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<tr>
<td>Points:</td>
<td>15</td>
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1. (15 points) Consider the points $A = (1, 2, 3)$, $B = (0, 2, 1)$, and $C = (4, 0, 3)$ in $\mathbb{R}^3$.

(a) Find a normal vector $\mathbf{n}$ to the plane $P$ containing the points $A, B, C$.

(b) Give an equation for the plane $P$.

(c) Find the area of the parallelogram spanned by the vectors $\mathbf{AB}$ and $\mathbf{AC}$.

(d) Consider the line $L$ given by the parametrization $\mathbf{r}(t) = (3 + t, -2 - t, 2)$. Is $L$ parallel to the plane $P$? Why or why not?
2. (6 points) Let $L$ be the line parametrized by $\mathbf{r}(t) = \mathbf{a} + t\mathbf{b}$ where $\mathbf{a} = \langle -3, 0, -3 \rangle$ and $\mathbf{b} = \langle 3, 2, 4 \rangle$. Find the point $Q$ of intersection of the line $L$ with the plane $3x - y + 2z = 15$.

3. (5 points) Find the midpoint $M$ of the straight-line segment between the points $P = (5, 2, 6)$ and $Q = (7, -1, 3)$. 
4. (18 points) For each function

(a) $x^2 - y^2$  
(b) $y \cos(x)$  
(c) $e^{-x^2-y^2}$

label its graph and its level set diagram from the options below. Here each level set diagram consists of level sets $\{f(x, y) = c_i\}$ drawn for evenly spaced $c_i$. 

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5. (15 points) EXACTLY one of the following limits exists. Circle the one that exists AND either evaluate it or prove the other limit does not exist.

\[ \lim_{(x,y) \to (0,0)} \frac{yx^2 + y^2x}{x^3 + y^3} \quad \text{and} \quad \lim_{(x,y) \to (0,0)} \frac{xy^4}{(x^2 + y^2)^2} \]

6. (8 points) Is \( u = e^{-4t} \sin(2x) \) a solution of the heat equation, that is, does \( u_t = u_{xx} \)?
7. (15 points) This question has two parts. Level curves are shown for a function \( f \).

(a) Determine whether the following are positive, negative, or zero at the point \( P \).

\[
\begin{array}{cccccc}
 f_x & f_y & f_{xx} & f_{yy} & f_{xy} & D_v(f) \\
\end{array}
\]

(b) Draw a vector in the direction of \( \nabla f(p) \) on the picture.

8. (10 points) Find an equation of the tangent plane to \( z = xy + \ln(x^2 + y^2) \) at \( (x, y) = (0, 1) \).
9. (8 points) Suppose $f$ is a differentiable function of $x$ and $y$, and
$g(u, v) = f(u^2 + \cos v, e^v + \sin u)$. Use the table of values to calculate $g_u(0, 0)$ and $g_v(0, 0)$.

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<tr>
<th></th>
<th>$f$</th>
<th>$g$</th>
<th>$f_x$</th>
<th>$f_y$</th>
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<tbody>
<tr>
<td>$(0,0)$</td>
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<td>2</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>$(1,1)$</td>
<td>4</td>
<td>2</td>
<td>5</td>
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