MATH 448, Complex Variables

Course Overview/Description
This course is for students who desire a rigorous introduction to the theory of functions of a complex variable. Topics include Cauchy’s theorem, the residue theorem, the maximum modulus theorem, Laurent series, the fundamental theorem of algebra, and the argument principle.

Course Goals
The course covers the first three chapters of the text *Complex Variables* by Stephen D. Fisher. Students should leave with a fundamental knowledge of the basic theory and applications of complex variables through reading, understanding and constructing proofs. It is important to recognize that reading ahead in the textbook before viewing class videos will make the lectures more comprehensible and writing your homework solutions in your own words improves your understanding.

Course Content

1. The Complex Plane
   - Complex numbers, complex plane
   - Triangle inequality
   - De Moivre’s Theorem
   - Algebraic construction of complex numbers
   - Geometry of the complex plane, topology of the complex plane
   - Functions and limits, infinite series
   - Exponential, logarithm, and trigonometric functions
   - Line integrals and Green’s theorem

2. Basic Properties of Analytic Functions
   - Complex differentiability
   - Analytic and harmonic functions; Cauchy-Riemann equations
   - Power series
   - Cauchy’s theorem and Cauchy’s formula
   - Consequences of Cauchy’s formula: The coefficient of power series, Liouville’s Theorem
   - The order of zero
   - Isolated singularities: removable singularities, poles, and essential singularities
   - Laurent series
   - Computation of residues
   - Residue theorem and evaluation of definite integrals and infinite sums

3. Analytic Functions as Mappings
   - Zeros of an analytic function
   - The Argument Principle
   - Rouché’s Theorem
   - The Fundamental Theorem of Algebra
   - Maximum Modulus Principle
   - Schwarz’s Lemma
   - Linear fractional transformations
Lecture Video Recording Info
This is an online course featuring video lectures from the course taught by Professor Bruce Reznick.

Textbook

Prerequisites
MATH 447 (NetMath course in development)

Requirements
Students must be able to print out assignments, write out solutions, then scan their written work and upload it to Moodle.