Math 444, Elementary Real Analysis

Course Description:
Math 444 is an introduction to $\varepsilon$-$\delta$ analysis on the real numbers which makes what the students have learnt from calculus courses rigorous. The course is for students who do not plan graduate study (Those students should take Math 447). Topics covered by Math 444 include the real number system, limits, continuity, derivatives, the Darboux integral and the Riemann integral, and sequences of functions.

Prerequisites: MATH 241 (Calculus III, Vector Calculus); MATH 347 (Fundamental Mathematics) or MATH 348 (Fundamental Mathematics), or equivalent

Course Goals:
Students should leave the course not only with a basic understanding of the fundamental concepts of real analysis, but also an improved ability at reading and writing mathematical arguments. Regular homework is an important aspect of the course.

Course Content Includes:

1. Preliminaries
   - Sets and Functions
   - Well-Ordering Property of Natural Numbers
   - Mathematical Induction
   - Cardinality
   - Cantor’s Theorem

2. The Real Numbers
   - Basic Properties of Real Numbers
   - Infimum and Supremum
   - The Completeness of Real Numbers
   - Archimedean Property
   - The density Theorem
   - Intervals

3. Sequences
   - Sequences and Limits
   - Monotone Sequences
   - Bolzano-Weierstrass Theorem
   - Limit Superior and Limit Inferior
   - The Cauchy Criterion
   - Infinite Series
   - The nth Term Test, Comparison Test, Limit Comparison Test
4. Limits
- Definition: Limits of Functions
- Sequential Criterion for Limits

5. Continuous Functions
- Continuous functions
- Sequential Criterion for Continuity
- Boundedness Theorem
- The Maximum-Minimum Theorem
- Bolzano’s Intermediate Theorem
- Uniform Continuity Theorem
- Lipschitz Function
- Continuous Extension Theorem
- Weierstrass Approximation Theorem

6. Differentiation
- The Derivative
- Caratheodory’s Theorem and Chain Rule
- The Derivative of Inverse Function
- Interior Extremum Theorem
- Rolle’s Theorem
- The Mean Value Theorem and Its Applications

7. The Riemann Integral
- Riemann Integral and Darboux Integral
- Riemann Integrable Functions
- Fundamental Theorem of Calculus

8. Sequences of Functions
- Pointwise and Uniform Convergence
- Cauchy Criterion for Uniform Convergence
- Interchange of a Function Limit

Course Format:

The course uses video lectures from Math 444 course taught by Professor Joseph Miles. The text for this course is Introduction to Real Analysis by Robert G. Bartle and Donald R. Sherbert, 4th edition and includes most of Chapters 1 through 7, plus Sections 8.1 and 8.2.

Math 444 is delivered using the Moodle (Learning Management System). Proctored exams are taken using pencil and paper.