Exercises of the form "Exercise a.b.c" refer to exercises in *Algebra: Abstract and Concrete* by Goodman, available for free from the link on the course website.

**Assignment**

1. Exercise 2.5.7
2. Exercise 2.5.8
3. Determine (with explanation) all subgroups of $D_4$.
4. (a) Show that the center $Z(G)$ is a normal subgroup of $G$.
   (b) Determine $Z(D_4)$.
5. Let $G$ be a group. Define $x \sim y$ if there is $g \in G$ such that $x = gyg^{-1}$. Show that $\sim$ is an equivalence relation.
6. (Extra Credit) A little bit of extra background beyond what we have discussed in class will help to to put this optional exercise into context.

Describing groups in terms of *generators* and *relations* is a useful for making computations in a group. For example, we described the dihedral group $D_n$ in terms of generators $r$ and $j$ and the relations $r^n = e$, $j^2 = e$, and $rj = j^{-1}r$. This information is displayed succinctly as

$$D_n = \langle r, j \mid r^n = j^2 = e, rj = j^{-1}r \rangle.$$

However, given a presentation of a group in terms of generators and relations, there may be “hidden relations”. To illustrate this subtlety, consider two groups, with very similar looking presentations:

- $X = \langle x, y \mid x^2 = y^2 = (xy)^2 = e \rangle$, and
- $Y = \langle s, t \mid s^3 = t^3 = (st)^3 = e \rangle$.

One can show that (this isn’t the exercise yet) $|X| = 4$ while $|Y| = \infty$.

Now for the exercise. Let $G$ be the group given in terms of generators and relations:

$$G = \langle a, b \mid a^4 = b^3 = e, ab = b^2a^2 \rangle.$$

Show that $G$ is the trivial group, i.e., that $G = \{e\}$. (*Hint:* proceed by deducing relations: (i) $a^3b = ba^3$, (ii) $ab = ba$, (iii) $ab = e$, and then finally that (iv) $a = e$, $b = e$.)