Topics

Material covered in class lectures 2/13 to 3/3. This corresponds to (parts of) Sections 2.3-2.7 in Goodman.

1. Be able to give precise and correct definitions of the important concepts from these lectures. Know the basic examples and properties.
   - dihedral groups
   - kernel of a homomorphism
   - normal subgroup
   - (left/right) coset
   - equivalence relation, equivalence class, partition
   - quotient group

2. Know what our important theorems say, know how and when to use them:
   - Lagrange’s theorem
   - the homomorphism theorem

3. Be able to prove results proved in class such as:
   - $\text{Ker}(f) = \{e\}$ if and only if $f$ is injective.
   - If $|G| = p$, where $p$ is a prime, then $G \cong \mathbb{Z}_p$.
   - The operation on $G/N$ is well-defined when $N$ is a normal subgroup of $G$.
   - If $\sim$ is an equivalence relation on $X$, then for any $x, y \in X$,
     (a) $x \sim y$ if and only if $[x] = [y]$.
     (b) Either $[x] = [y]$ or $[x] \cap [y] = \emptyset$.

Practice problems

Review homework problems. An old exam is available on the course canvas site. Here are a few further practice problems. Here are a few further practice problems.

**Question 1** Consider the subgroups of $D_5$, $H = \langle r \rangle$ and $K = \langle j \rangle$. Are these normal subgroups?

**Question 2** Let $G$ be a group. Consider the relation on $G$ defined by $g \sim h$ if there is an element $k \in G$ such that $g = khk^{-1}$. Show that $\sim$ is an equivalence relations.

**Question 3** Suppose that $d | n$. Show that $f : \mathbb{Z}_n \to \mathbb{Z}_d$ defined by $f([a]_n) = [a]_d$ is a well-defined homomorphisms. What is its kernel? What is $\mathbb{Z}_n/\text{Ker}(f)$?

**Question 4** Is $\mathbb{Z}_9 \to \mathbb{Z}_5$, $[a]_9 \mapsto [a]_5$ well-defined?

**Question 5** Is $SO_n \leq O_n$ normal? If so what is $O_n/SO_n$? Is $O_n \leq GL_n$ normal? If so, what is $GL_n/O_n$?

**Question 6** Does the formula $aH \cdot bH = abH$ define a group structure on the set of cosets $G/H$ for:
   (a) $G = S_3$, $H = \langle (1 2) \rangle$
   (b) $G = S_3$, $H = \langle (1 3 2) \rangle$
**Question 7** How many elements does the subgroup of even permutations in $S_n$ have?

**Question 8** Show that $\mathbb{Z}_{10}$ is isomorphic to $\mathbb{Z}_2 \times \mathbb{Z}_5$.

**Question 9** Lagrange’s theorem tells us that the only possible orders of elements of $G$ are the divisors of $|G|$. Let $d$ be a divisor of 18. Does $\mathbb{Z}_{18}$ always have an element of order $d$? Does $D_9$ always have an element of order $d$?