Question 1
Let $G$ be a group. An automorphism of $G$ is an isomorphism $f : G \to G$. Write $\text{Aut}(G)$ for the set of automorphisms of $G$.
(a) Check that $\text{Aut}(G)$ is a group.

(b) Write $c_g : G \to G$ for conjugation by $g$, so $c_g(x) = gxg^{-1}$. Check that

$$c : G \to \text{Aut}(G), \ g \mapsto c_g$$

is a homomorphism.

Question 2  Lets consider the automorphisms of $S_3$.
(a) Any automorphism of $S_3$ permutes the set of 2-cycles $\{(1\ 2), (1\ 3), (2\ 3)\}$. Why?

(b) Any automorphism of $S_3$ is completely determined by what its values on the 2-cycles. Why?

(c) $|\text{Aut}(S_3)| \leq 6$. Why?
Question 3  The homomorphism $c : G \to \text{Aut}(G)$ is not always surjective and its image $c(G)$ in $\text{Aut}(G)$ is called the group of inner automorphisms of $G$, written $\text{Int}(G)$:

$$\text{Int}(G) := c(G) \leq \text{Aut}(G).$$

Recall that the center of $G$ is defined by $Z(G) = \{ g \in G \mid gx = xg \text{ for all } x \in G \}$.

(a) Show that

$$G/Z(G) \cong \text{Int}(G).$$

(b) Is every automorphism of $S_3$ inner? (Why/Why not?)