Problem 1 The luminosity of certain giant and supergiant stars varies in a periodic manner. It is hypothesized that the period \( p \) depends upon the star’s average radius \( r \), its mass \( m \), and the gravitational constant \( G \).

1. Newton’s law of gravitation asserts that the attractive force between two bodies is proportional to the product of their masses divided by the square of the distance between them, that is
   \[
   F = \frac{G m_1 m_2}{d^2},
   \]
   where \( G \) is the gravitational constant. From this determine the (fundamental) dimensions of \( G \).

2. Use dimensional analysis to determine the functional dependence of \( p \) on \( m \), \( r \), and \( G \).

3. Arthur Eddington used the theory for thermodynamic heat engines to show that
   \[
   p = \sqrt{\frac{3\pi}{2},}
   \]
   where \( \rho \) is the average density of the star and \( \gamma \) is the ratio of specific heats for stellar material. How does this differ from your result?

Problem 2 Holmes 1.6

The frequency \( \omega \) of waves on a deep ocean is found to depend on the wavelength \( \lambda \) of the wave, the surface tension \( \sigma \) of the water, the density \( \rho \) of the water, and gravity.

1. Use dimensional reduction to determine the functional dependence of \( \omega \) on \( \lambda, \sigma, \rho, \gamma \).

2. In fluid dynamics it is shown that
   \[
   \omega = \sqrt{\frac{g k}{\sigma + \frac{k^3}{\rho}}},
   \]
   where \( k = 2\pi/\lambda \) is the wavenumber. How does this differ from your result?

Problem 3 Holmes 1.8

The equations that account for the relativistic motion of a planet around the sun are

\[
\frac{d^2 r}{dt^2} - r \left( \frac{d\theta}{dt} \right)^2 = -\frac{G m}{r^2} + \frac{b}{r^3},
\]

\[
\frac{d}{dt} \left( r^2 \frac{d\theta}{dt} \right) = 0.
\]

where \( b \) is a constant.

Assume the initial conditions are \( r = r_0, r' = 0, \) and \( \theta = 0 \) at \( t = 0 \).

1. What are the dimensions of \( r_0 \), \( b \)?

2. Nondimensionalize the problem. The scaling should be chosen so the only nondimensional group appearing in the problem involves \( b \).

Problem 4 Assume that we consider only length, mass, and time as fundamental units. In each of the following cases, explain whether or not we can use the triple as an equivalent set of fundamental units:
1. density, volume, velocity
2. density, velocity, acceleration
3. volume, velocity, acceleration
4. \([c], [h], [G]\), where \(c\) is the speed of light, \(h\) is Planck’s constant, and \(G\) is the gravitational constant

**Problem 5** Consider the polynomial

\[ p_W(x) = \prod_{i=1}^{20} (x - i) \]  

obviously the roots are distinct, and given by \(x = i \in (1 \ldots 20)\). Now consider the polynomial

\[ \tilde{p}_W(x) = p_w(x) + 10^{-8} x^{19} \]

1. What is the percentage change in the coefficient of \(x^{19}\).

2. Use Matlab, Mathematica or a similar package to numerically compute the roots. How do they compare with the unperturbed roots \(i \in (1 \ldots 20)\)

3. Is this a small perturbation?