1. \[ F = G \frac{m_1 m_2}{d^2}, \quad G = \frac{F d^2}{m_1 m_2} \]

\[ [G] = \frac{kg \cdot M \cdot m^2}{s^2 (kg)^2} = \frac{m^3}{s^2 kg} \]

\[ [F] = N \]

\[ [m] = kg \]

\[ [r] = m \]

Dimensionless quantity

\[ \varepsilon_1 = \frac{G \rho^2 m}{r^3} \]

Note: density

\[ \rho = \frac{m}{\frac{4}{3} \pi r^3} \]

\[ \varepsilon_1 = G \rho^2 \frac{4}{3} \pi \rho \]

by Buckingham

\[ \rho^* = \alpha \sqrt[3]{\frac{3}{4\pi \rho G}} \]
Thus dimensional analysis recovers Eddington’s result up to the constant $\alpha$.

The correct constant $\alpha = \frac{\sqrt{\pi}}{\sqrt{\gamma}}$ cannot be determined without some physics.

2. $[1] = m$  $[\omega] = \frac{kg}{m \cdot sec^2}$  $[\rho] = \frac{kg}{m^3}$

$[g] = \frac{m}{sec^2}$  $[\omega] = \frac{1}{sec}$

Note that there are two dimensionless quantities.

$E_1 = \frac{\omega^2}{g}  \quad \quad E_2 = \frac{\sigma}{\rho g a^2}$

by Buckingham II theorem

$E_i = \phi_i(E_2)$

$\omega^2 = \frac{g}{a} f \left( \frac{\sigma}{\rho g a^2} \right)$

Taking $f(E_2) = 2\pi + \frac{8\pi^2}{3} E_2$ gives

$\omega^2 = \frac{g^2 \pi}{a} + \frac{\sigma}{\rho \pi \frac{8\pi^2}{3}}$

$\omega = \sqrt{\frac{8\pi}{\rho} \frac{\sigma}{\rho}}$

The correct form of $\omega$ cannot be found by D.A.
\[ [c] = m \quad [r] = m \quad [b] = \frac{m^4}{sec^2} \quad [Gm] = \frac{m^3}{sec^2} \quad [zz] = sec \]

Dimensionless groups

\[ E_1 = \frac{r_0}{r} \quad E_2 = \frac{Gm \, r_0^2}{b} \]

Define \( \beta \)

\[ E_3 = \frac{Gm}{r_0^3} \]

\[ P = \frac{r_0}{r} \quad S = 2 \sqrt{\frac{Gm}{r_0^3}} \]

(Nondimensional Distance) (Non-dimensional Time)

\[ r = r_0 \rho \quad ds = \rho \, dt \sqrt{\frac{Gm}{r_0^3}} \quad dt = \sqrt{\frac{r_0^3}{Gm}} \]

Eqs. become

\[ \frac{Gm \, \beta}{r_0^3} \frac{d^2 \rho}{ds^2} - \frac{r_0 \, \rho \, (\frac{d \beta}{ds})^2}{\beta} = -\frac{Gm \, \beta}{r_0^2 \rho^2} + \frac{b}{r_0^3 \rho^3} \]

\[ \frac{d^2 \rho}{ds^2} - \rho \left( \frac{d \beta}{ds} \right)^2 = -\frac{1}{\rho^2} + \frac{b}{Gm \, \beta \, \rho^3} \frac{1}{r_0^3} \]

\[ \frac{d}{ds} \left( \rho^2 \frac{d \theta}{ds} \right) = 0 \quad \frac{1}{E_2} \]

\( \rho = 1 \), \( \rho' = 0 \) and \( \theta = 0 \) at \( t = 0 \)
4. \[ \frac{m}{L^3}, \frac{L}{T}, \frac{L}{T^2} \] Yes. \[ \begin{pmatrix} 1 & 0 & 0 \\ -3 & 0 & 0 \\ 0 & 3 & -1 \end{pmatrix} \text{ are lin ind.} \]

5. \[ \frac{m}{L^3}, \frac{L}{T}, \frac{L}{T^2} \] No - No mass unit possible

6. \[ \frac{L}{T}, \frac{M}{L^3}, \frac{L^3}{T^2} \] \[ \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ -1 & -1 & 3 \end{pmatrix} \] \[ \text{Yes - lin independent} \]

5) \[ P_{\text{v}} = \frac{20}{11}(X_0 - i) = X - \sum_{i=1}^{20} X_i \]

\[ = X - 210X \]

Adding 16 \[ X_0 \] changes this coeff by 4.7 x 10⁻⁹.

First 16 (i...16) change by less than 1%.

11 and 12 move by a lot and (0.25 and 0.75) and are almost degenerate.

The roots of the equator collide and form complex conjugate pairs.

For the first 8-10 roots the perturb is small. For the rest the perturb is not small.