A review of some important calculus topics

1. Chain Rule:
   
   (a) Let \( h(t) = \sin(\cos(t \tan t)) \). Find the derivative with respect to \( t \).
   
   (b) Let \( s(x) = \sqrt[3]{x} \) where \( x(t) = \ln(f(t)) \) and \( f(t) \) is a differentiable function. Find \( \frac{ds}{dt} \).

2. Parameterized curves:

   (a) Describe and sketch the curve given parametrically by
   
   \[
   \begin{align*}
   x &= 5 \sin(3t) \\
   y &= 3 \cos(3t)
   \end{align*}
   \]
   
   for \( 0 \leq t < \frac{2\pi}{3} \).

   What happens if we instead allow \( t \) to vary between 0 and \( 2\pi \)?

   (b) Set up, but do not evaluate an integral that calculates the arc length of the curve described in part (a).

   (c) Consider the equation \( x^2 + y^2 = 16 \). Graph the set of solutions of this equation in \( \mathbb{R}^2 \) and find a parameterization that traverses the curve once counterclockwise.

3. 1st and 2nd Derivative Tests:

   (a) Use the 2nd Derivative Test to classify the critical numbers of the function \( f(x) = x^4 - 8x^2 + 10 \).

   (b) Use the 1st Derivative Test and find the extrema of \( h(s) = s^4 + 4s^3 - 1 \).

   (c) Explain why the 2nd Derivative test is unable to classify all the critical numbers of \( h(s) = s^4 + 4s^3 - 1 \).

4. Consider the function \( f(x) = x^2 e^{-x} \).

   (a) Find the best linear approximation to \( f \) at \( x = 0 \).

   (b) Compute the second-order Taylor polynomial at \( x = 0 \).

   (c) Explain how the second-order Taylor polynomial at \( x = 0 \) demonstrates that \( f \) must have a local minimum at \( x = 0 \).

5. Consider the integral \( \int_0^{\sqrt{3\pi}} 2x \cos(x^2) \, dx \).

   (a) Sketch the area in the \( xy \)-plane that is implicitly defined by this integral.

   (b) To evaluate, you will need to perform a substitution. Choose a proper \( u = f(x) \) and rewrite the integral in terms of \( u \). Sketch the area in the \( uv \)-plane that is implicitly defined by this integral.

   (c) Evaluate the integral \( \int_0^{\sqrt{3\pi}} 2x \cos(x^2) \, dx \).