1. Salmonella bacteria, found on almost all chicken and eggs, grow rapidly in a nice, warm place. If just a few hundred bacteria are left on the cutting board when a chicken is cut up, and they get into the potato salad, the population begins compounding! Suppose the number present in the potato salad after $x$ hours is given by:

$$f(x) = 500 \times 2^{3x}$$

a) If the potato salad is left out on the table, how many bacteria are present 1 hour later?

b) How many were present initially?

c) How often do the bacteria double?

d) How quickly will the number of bacteria increase to 32,000?
2. A company must pay a $307,000 settlement in 3 years.

(a) What amount must be deposited now, compounded at 6% semi-annually, to have enough money for the settlement?

(b) How must interest will be earned?

(c) Suppose the company can deposit only $200,000 now. How much more will be needed in 3 years?

(d) Suppose that the company can deposit $200,000 now in an account that pays interest continuously. What interest rate would they need to accumulate the entire $307,000 in 3 years?
3. You are offered 2 jobs starting July 1, 2013. Humongous Enterprises offers you $45,000 a year to start, with a raise of 4% every July 1. At Crabapple Inc., you start with $30,000, with an annual increase of 6% every July 1. On July 1 of what year would the job at Crabapple Inc. pay more than the job at Humongous Enterprises? Solution: 2035

4. Lucky Larry was faced with solving

\[ \log(2x + 1) - \log(3x - 1) = 0 \]

Larry just dropped the logs and proceeded:

\[ (2x + 1) - (3x - 1) = 0 \]
\[ -x + 2 = 0 \]
\[ x = 2 \]

Although Lucky Larry is wrong in dropping the logs, his procedure will always give the correct answer to an equation of the form

\[ \log(A) - \log(B) = 0 \]

where \( A \) and \( B \) are two expressions in \( x \). Prove that \( \log(A) - \log(B) = 0 \) implies the equation \( A - B = 0 \), which is what you get when you drop the logs.

(Hint: Start with the equation \( \log(A) - \log(B) = 0 \), then use log rules to rearrange this equation until you get \( A - B = 0 \))
Recall: \( b^{\log_b(x)} = x \) and in particular \( e^{\ln(x)} = x \)

5. Using the properties/rules of logarithms, you will now prove that \( e^{\ln(x)} = x \) by filling in the table below.

We want to find the value \( y \) of the following expression

\[ e^{\ln(x)} = y \]

<table>
<thead>
<tr>
<th>Step</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Start with the original equation</td>
<td>( e^{\ln(x)} = y )</td>
</tr>
<tr>
<td>Take the natural log of both sides of the equation</td>
<td></td>
</tr>
<tr>
<td>Now use the power rule for logs on the left-hand side to bring down</td>
<td></td>
</tr>
<tr>
<td>the exponent</td>
<td></td>
</tr>
<tr>
<td>Now simplify using ( \ln(e) = 1 )</td>
<td>( \ln(x) = \ln(y) )</td>
</tr>
<tr>
<td>Now use the equality rule for logs</td>
<td>( x = y )</td>
</tr>
</tbody>
</table>

So we conclude \( e^{\ln(x)} = x \)

6. Suppose you want to prove the rule \( b^{\log_b(x)} = x \) following the steps above. First, you write \( b^{\log_b(x)} = y \). Now, your next step differs from the work above! Instead of taking the natural log of both sides, what should you do, and why?