Exponent and Logarithm Review

For \( a > 0, \ a \neq 1, \) and \( x > 0 \) we define the logarithm function to be:

\[ y = \log_a(x) \text{ means } a^y = x \]

We choose this definition to create a function which is the inverse to the exponential function.

1. Find \( y = \log_4(64) \)

   \( y = 3 \) works since \( 4^3 = 64 \).

   So \( \log_4(64) = 3 \).

2. Use the definition of logarithm to find \( \log_8(64) \) and \( \log_3\left(\frac{1}{9}\right) \)

Properties of the Logarithm Function

\( \log_e(x) \) is abbreviated as \( \ln(x) \) \quad \( \log_{10}(x) \) is abbreviated as \( \log(x) \)

\[
\begin{align*}
\log_a(xy) &= \log_a(x) + \log_a(y) \\
\log_a\left(\frac{x}{y}\right) &= \log_a(x) - \log_a(y) \\
\log_a(x^r) &= r \cdot \log_a(x) \\
\log_a(a) &= 1 \\
\log_a(1) &= 0 \\
\log_a(a^r) &= r
\end{align*}
\]
3. Use the properties of logarithms above to expand each expression as a sum, difference, or product of simpler logarithms.

(a) \( \log_5(3k) \)

(b) \( \log_7 \left( \frac{15p}{7y} \right) \)

4. Suppose \( \log_b(2) = a \) and \( \log_b(3) = c \). Find the following:

(a) \( \log_b(32) \)

**Solution:**
\[ \log_b(32) = \log_b(2^5) = 5 \cdot \log_b(2) = 5a \]

(b) \( \log_b(18) \)
(c) \( \log_b (72b) \)

(d) \( \log_b (9b^2) \)

Change of Base Formula

\[
\log_a(x) = \frac{\log_b(x)}{\log_b(a)} = \frac{\ln(x)}{\ln(a)}
\]

5. Use your calculator and the change of base formula to evaluate:

(a) \( \log_5(80) \)

(b) \( \log_{12}(210) \)
Solving Logarithmic Equations

6. Solve $log_{x}(\frac{8}{27}) = 3$ for $x$.

   Write the logarithm in exponential form $x^3 = \frac{8}{27}$
   Take the 3rd root of both sides $x = \frac{2}{3}$

7. Solve the following for $x$:

   (a) $log_2(x) - log_2(x - 1) = 1$

   (b) $log_2(x^2 - 1) = log_2(x + 1)$

   (c) $log_8(16) = x$
Solving Exponential Equations

8. Solve $3^x = 5$.

Take the natural logarithm of both sides

\[ \ln(3^x) = \ln(5) \]

Use a property of logarithms to bring the $x$ down

\[ x \cdot \ln(3) = \ln(5) \]

Divide both sides by $\ln(3)$

\[ x = \frac{\ln(5)}{\ln(3)} \]

Evaluate in your calculator

\[ x \approx 1.465 \]

9. Use properties of logarithms to solve the following:

(a) $3^{2x} = 4^{x+1}$

(b) $5e^{0.01x} = 9$
10. Allison Andrews invests $15,000 in an account paying 7% per year compounded annually. In how many years will the amount at least triple?

11. Andrea Davis plans to invest $600 into a money market account. Find the interest rate that is needed for the money to grow to $1240 in 14 years if the interest is compounded continuously.