Math 234 Spring 2017 Exam 1 Version 1
Wednesday, February 15th, 2017

Name: ________________________________ UIN: __________________________

Circle the section you are registered for:

DDA (Artur 9AM)  DDB (Artur 10AM)  DDC (Lutian 11AM)  DDD (Lutian 12PM)
DDE (Shiyu 1PM)  DDF (Bill 2PM)  DDG (Bill 3PM)  DDH
BDA (Paulina 9AM)  BDB (Paulina 10AM)  BDC (Alessandro 11AM)  BDD (Alessandro 12PM)
BDE (Julian 1PM)  BDF (Dileep 2PM)  BDG (Dileep 3PM)  BDH (Julian 10AM)
CDA  CDB (Mingyu 11AM)  CDC (Dara 12PM)  CDD (Mingyu 1PM)
CDE (Dara 2PM)  CDF (Shiyu 3PM)  CDG (Chris 11AM)  CDH

(1) No baseball caps, hoodies, etc. or dark sunglasses. All hats are to be removed.

(2) All bags, purses, jackets, and other personal items should be at the front of the room.

(3) If you have a question, raise your hand and a proctor will come to you. If you have
to use the bathroom, do so NOW. You will not be permitted to leave the room and
return during the exam.

(4) No cells phones, i-Pods, MP3 players. Turn them off now. If you are seen these items
in hand during the exam it will be considered cheating and you will be asked to leave.
This includes using it as a time-piece.

(5) Be sure to print your proper name clearly at the top of this page. Also circle the
section for which you are registered.

(6) If you finish early, quietly and respectfully get up and hand in your exam.

(7) When time is up, you will be instructed to put down your writing utensil, close your
exam and remain seated. Anyone seen continuing to write after time is called will
have their exam marked and lose all points on the page they are writing on.

(8) To ensure that you receive full credit, show all of your work.

(9) Good luck. You have 60 minutes to complete this exam.
(1) (12 points) A grocery store owner marks down the supply and demand functions for pumpkins each fall.

\[ p = D(q) = 121 - q \]
\[ p = S(q) = (1/10)q \]

(a) (2 points) If he wants to price pumpkins at $5 each, how many can he expect to sell?

\[ p = D(q) = 121 - q \]
\[ 5 = 121 - q \]
\[ q = 121 - 5 \Rightarrow q = 116 \]

(b) (2 points) If he prices pumpkins at $5 each, how many pumpkins can he expect to be supplied to him?

\[ p = S(q) = \frac{1}{10}q \]
\[ 5 = \frac{1}{10}q \Rightarrow q = 50 \]

(c) (2 points) At the price of $5, is there a shortage or a surplus of pumpkins?

Shortage.

because at $5, supply < demand.

(d) (6 points) Find the equilibrium price and the equilibrium quantity for these supply and demand functions.

Set \( D(q) = S(q) \)

\[ 121 - q = \frac{1}{10}q \]

\[ 121 = q + \frac{1}{10}q \]

\[ 121 = \frac{11}{10}q \]

\[ q = \frac{10}{11} \times 121 \]

\[ q = 110 \]

plug in supply function

\[ P = \left(\frac{1}{10}\right)(110) \]

\[ P = \$11 \]
(2) (10 points) A boutique can obtain a handbag from a distributor at the cost of $5 per bag. The store has been selling the bag for $40 each, and at this price selling 50 bags a month. The boutique is planning to lower the price to stimulate sales, and it estimates that for each $1 reduction in price, 5 more handbags will be sold. Let \( x \) represent the number of $1 reductions in price.

In terms of \( x \), write the equations for the monthly cost \( C(x) \), revenue \( R(x) \), and profit \( P(x) \) from the sale of this handbag.

\[
X = \# \text{ of } $1 \text{ price reduction}
\]

\[
C(X) = (\text{price per item})(\# \text{ of items sold}) = 5(50 + 5x)
\]

\[
R(X) = (\text{price per item})(\# \text{ of items sold}) = (40 - x)(50 + 5x)
\]

\[
P(X) = R(x) - C(x) = (40 - x)(50 + 5x) - 5(50 + 5x)
\]

(3) (8 points) Delaney deposits $1,000 in a savings account, at 5% interested, compounded semi-annually (twice a year).

(a) (6 points) What is the effective interest rate?

\[
\gamma_E = \left(1 + \frac{r}{m}\right)^m - 1 = \left(1 + \frac{0.05}{2}\right)^2 - 1
\]

(b) (2 points) Delaney wants to leave her money in the account until it grows to at least $2,000. She sets up the following equation and solves for \( t \):

\[
A = P(1 + r/m)^{mt}
\]

\[
t = 14.03551...
\]

How long should Delaney leave her money in the account for?

14.5 years
(4) (10 points) Consider the piece-wise defined function

\[ f(x) = \begin{cases} 
3x^2 + 2x & \text{if } x < 0 \\
mx & \text{if } x \geq 0
\end{cases} \]

where \( m \) is a constant, real number.

(a) (3 points) Is \( f(x) \) continuous at \( x = 0 \)? Justify your answer.

\[ \begin{align*}
\text{LHL} &= \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (3x^2 + 2x) = 3(0)^2 + 2(0) = 0 \\
\text{RHL} &= \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (mx) = m(0) = 0
\end{align*} \]

So \( f \) is continuous at \( x = 0 \).

(b) (3 points) Compute \( \lim_{h \to 0} \frac{f(h) - f(0)}{h} \). Show your work.

\[ \begin{align*}
\lim_{h \to 0^-} & \frac{3h^2 + 2h - m(0)}{h} \\
&= \lim_{h \to 0^-} \frac{3h^2 + 2h}{h} = \lim_{h \to 0^-} \frac{h(3h + 2)}{h} = \lim_{h \to 0^-} (3h + 2) = 3(0) + 2 \\
&= 2
\end{align*} \]

(c) (3 points) Compute \( \lim_{h \to 0^+} \frac{f(h) - f(0)}{h} \). Show your work.

\[ \begin{align*}
\lim_{h \to 0^+} & \frac{f(h) - f(0)}{h} = \lim_{h \to 0^+} \frac{mh - m(0)}{h} = \lim_{h \to 0^+} \frac{mh}{h} = \lim_{h \to 0^+} (m) = m \\
&\text{\scriptsize{\textcolor{red}{\text{Because } m \text{ is a constant}}}}
\end{align*} \]

(d) (1 points) Find the value of \( m \) so that \( f'(0) \) exists. Justify your answer.

\[ m = 2 \]
(5) (15 points) Alice the biologist is studying the science behind bread-making. In addition to metabolizing starches into CO₂ gas for the bread to rise, the yeast also grow in number. In one particular experiment, she observes that the amount (in grams) of yeast is
\[ f(t) = 3 \times (16)^{t/12} \]
where \( t \) denotes hours after 12pm.

Write all of the answers in EXACT FORM (e.g. write \( \ln(2) \) rather than its decimal approximation).

(a) (5 points) How much yeast is present at 3pm?

\[ f(3) = 3 \times (16)^{3/12} \]
\[ = 3 \times 16^{1/4} \]
\[ = 3 \times (2)^{4/4} \]
\[ = 3 \times 2 = 6 \text{ grams} \]

(b) (10 points) When will the amount of yeast reach 24 grams? Show your work.

\[ 24 = 3 \times (16)^{t/12} \]
\[ \frac{24}{3} = (16)^{t/12} \]
\[ 8 = (16)^{t/12} \]
\[ \ln 8 = \ln (16)^{t/12} \]
\[ \ln 8 = \frac{t}{12} \ln (16) \]
\[ t = \frac{12 \ln 8}{\ln (16)} \]

(6) (10 points) Alice runs an experiment with a new strain of yeast to see how quickly it grows and multiplies. She starts with 3 grams of yeast, and observes it grows to 7 grams in 14 hours. Assuming exponential growth, find the growth function for the yeast (show all work).

\[ Y(t) = 3 e^{kt} \]

\[ 7 = 3 e^{14k} \]
\[ \frac{7}{3} = e^{14k} \]
\[ \ln \left( \frac{7}{3} \right) = 14k \ln (e) \]
\[ \ln \left( \frac{7}{3} \right) = 14k \]
\[ k = \frac{\ln \left( \frac{7}{3} \right)}{14} \]

Therefore, growth function is
\[ Y(t) = 3 e^{\frac{\ln \left( \frac{7}{3} \right)}{14} t} \]
(7) (20 points) Suppose that the derivative of \( f(x) = \sqrt{x^2 + 9} \) has already been computed to be
\[
f'(x) = \frac{x}{\sqrt{x^2 + 9}}
\]

(a) (10 points) Find the equation of the tangent to the graph of \( y = f(x) \) at \((4, f(4))\).

\[
\text{Slope } m = f'(4) = \frac{4}{\sqrt{16+9}} = \frac{4}{\sqrt{25}} = \frac{4}{5}
\]

\[
\text{Y-Coordinate } = f(4) = \sqrt{16+9} = \sqrt{25} = 5
\]

\[
\text{Eq. of tangent line is } y - y_0 = m(x-x_0),
\]

\[
y - 5 = \frac{4}{5}(x - 4)
\]

(b) (3 points) What is the instantaneous rate of change of the function when \( x = 1 \)?
\[
f'(1) = \frac{1}{\sqrt{1^2+9}} = \frac{1}{\sqrt{10}}
\]

(c) (7 points) Find all points on the graph where the tangent line is horizontal. List the points as \((x, y)\).

Set \( f'(x) = 0 \)
\[
\frac{x}{\sqrt{x^2 + 9}} = 0
\]

\[
\Rightarrow x = 0
\]

\[
\text{Y-Coordinate is } f(0) = \sqrt{0^2 + 9} = 3
\]

Therefore, only one point \((0, 3)\) where the tangent line is horizontal.
(8) (15 points) Alice the biologist is studying the panda population. She observes that the panda population is

\[ f(t) = \frac{1000 + 2500t + 8000t^2}{(2 + 2t)^2} \]

where \( t \) denotes the number of years after 2017.

(a) (3 points) What is the panda population in 2017? Show your work.

put \( t = 0 \) in above formula:

\[ f(0) = \frac{1000 + 2500(0) + 8000(0)^2}{(2 + 2(0))^2} = \frac{1000}{4} = \boxed{250} \]

(b) (10 points) Compute \( \lim_{t \to \infty} f(t) \). Show your work.

\[
\begin{align*}
\lim_{t \to \infty} f(t) &= \lim_{t \to \infty} \frac{1000 + 2500t + 8000t^2}{4 + 4t^2 + 8t} \\
&= \lim_{t \to \infty} \frac{1000 + 2500t + 8000t^2}{4 + 4t^2 + 8t} \cdot \frac{\frac{1}{t^2}}{\frac{1}{t^2}} \\
&= \lim_{t \to \infty} \frac{\frac{1000}{t^2} + \frac{2500}{t} + \frac{8000}{1}}{\frac{4}{t^2} + \frac{4}{t} + \frac{8}{t^2}} \\
&= \frac{\frac{1000}{\infty} + \frac{2500(0)}{\infty} + \frac{8000}{1}}{\frac{4(0)}{\infty} + \frac{4(0)}{\infty} + \frac{8(0)}{\infty}} = \frac{8000}{4} = \boxed{2000} \end{align*}
\]

(c) (2 points) In the long term, do the pandas become extinct or not? Justify your answer.

No, in the long term, the panda population approaches 2,000.
(9) (BONUS - EXTRA CREDIT) Let \( f(x) = \sqrt{2x + 5} \). Compute \( f'(x) \) using the limit definition of the derivative. Show your work.

\[
f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}
\]

\[
= \lim_{h \to 0} \frac{\sqrt{2(x+h) + 5} - \sqrt{2x + 5}}{h}
\]

\[
= \lim_{h \to 0} \frac{\sqrt{2x+2h+5} - \sqrt{2x+5}}{h}
\]

\[
= \lim_{h \to 0} \frac{\sqrt{2x+2h+5} - \sqrt{2x+5}}{h} \cdot \frac{\sqrt{2x+2h+5} + \sqrt{2x+5}}{\sqrt{2x+2h+5} + \sqrt{2x+5}}
\]

\[
= \lim_{h \to 0} \frac{(\sqrt{2x+2h+5})^2 - (\sqrt{2x+5})^2}{h(\sqrt{2x+2h+5} + \sqrt{2x+5})}
\]

\[
= \lim_{h \to 0} \frac{(2x+2h+5) - (2x+5)}{h(\sqrt{2x+2h+5} + \sqrt{2x+5})}
\]

\[
= \lim_{h \to 0} \frac{2h}{h(\sqrt{2x+2h+5} + \sqrt{2x+5})}
\]

\[
= \lim_{h \to 0} \frac{2}{\sqrt{2x+2h+5} + \sqrt{2x+5}} = \frac{2}{\sqrt{2x+5} + \sqrt{2x+5}} = \frac{2}{2\sqrt{2x+5}}
\]