Circle your TA discussion section.

- BDA, TR 09:00-09:50, Felix Clemen
- BDB, TR 10:00-10:50, Felix Clemen
- BDC, TR 11:00-11:50, Kexin Jin
- BDD, TR 12:00-12:50, Weihang Wang
- BDE, TR 01:00-01:50, Qiang Wu
- BDF, TR 02:00-02:50, Travis Nell
- BDG, TR 03:00-03:50, Cameron Rudd
- BDH, TR 09:00-09:50, Venkata Sai Narayana
- BDI, TR 10:00-10:50, Venkata Sai Narayana
- BDJ, TR 11:00-11:50, Lutian Zhao
- BDK, TR 12:00-12:50, Jose Gabriel
- BDL, TR 01:00-01:50, Weihang Wang
- BDM, TR 02:00-02:50, Cameron Rudd
- BDN, TR 03:00-03:50, Travis Nell
- BDO, TR 10:00-10:50, Anh Tuong Nguyen
- BDP, TR 11:00-11:50, Michael Duffy
- BDQ, TR 12:00-12:50, Qiang Wu
- BDR, TR 01:00-01:50, Madina Bolat
- BDS, TR 02:00-02:50, Kexin Jin
- BDT, TR 11:00-11:50, Jose Gabriel
- BDU, TR 09:00-09:50, Anh Tuong Nguyen
- BDV, TR 10:00-10:50, Michael Duffy
- BDW, TR 03:00-03:50, Madina Bolat

- Do not open this test booklet until you are told to do so.
- Turn off all electronic devices and put away all items except a pen/pencil and an eraser.
- No baseball caps, hoodies, etc. or dark sunglasses. All hats are to be removed.
- There is no partial credit on multiple-choice questions. For all other questions, you must show sufficient work to justify your answer.
- While the test is in progress, we will not answer questions concerning the test material.
- If you finish early, quietly and respectfully get up and hand in your exam.
- When time is up, you will be instructed to put down your writing utensil, close your exam and remain seated. Anyone seen continuing to write after time is called will have their exam marked and lose all points on the page they are writing on.
- Good luck. You have 60 minutes to complete this exam.

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1. (5 points) Find the derivative of \( f(x) = \frac{4}{x^3} + e^{4x} + \ln(x^3 + 3) \).

\[
\frac{df}{dx} = \frac{d}{dx} \left( 4x^{-3} + e^{4x} + \ln(x^3 + 3) \right)
\]

\[
\frac{df}{dx} = 4 \frac{d}{dx}(x^{-3}) + \frac{d}{dx}(e^{4x}) + \frac{d}{dx}\ln(x^3 + 3)
\]

\[
= (4)(-3x^{-4}) + 0 + \frac{1}{x^3 + 3} \frac{d}{dx}(x^3 + 3)
\]

\[
\frac{df}{dx} = -12x^{-4} + \frac{1}{x^3 + 3}(2x)
\]

2. (5 points) Find the derivative of \( f(x) = \frac{x^2 + 7x}{2x^3 + 3} \).

\[
f'(x) = \frac{\frac{d}{dx}(x^2 + 7x)(2x^3 + 3) - (x^2 + 7x)\frac{d}{dx}(2x^3 + 3)}{(2x^3 + 3)^2}
\]

\[
f'(x) = \frac{(2x + 7)(2x^3 + 3) - (x^2 + 7x)(6x)}{(2x^3 + 3)^2}
\]
3. (10 points) Find $f'(x)$ given that $f(x) = e^{(x^2 + 2x)^{3/2}}$

$$U = (x^2 + 2x)^{3/2}.$$ 

$$f = e^u.$$ 

$$\frac{df}{dx} = \frac{df}{du} \cdot \frac{du}{dx}.$$ 

$$\frac{df}{dx} = e^u \cdot \frac{2}{3} (x^2 + 2x)^{1/2} \cdot \frac{d}{dx} (x^2 + 2x).$$ 

$$= e^u \cdot \frac{2}{3} (x^2 + 2x)^{1/2} \cdot 2x.$$ 

$$= e^{(x^2 + 2x)^{3/2}} \cdot 2x (x + 1).$$

4. (10 points) Find an equation of the line tangent to the graph of $f(x) = x\sqrt{x^3 + 1}$ at the point $(2, 6)$.

$$f'(x) = \frac{d}{dx} \left( x \cdot \sqrt{x^3 + 1} \right)$$

$$f'(x) = \frac{d}{dx} (x) \sqrt{x^3 + 1} + x \frac{d}{dx} \left( \sqrt{x^3 + 1} \right)$$

$$f'(x) = \sqrt{x^3 + 1} + x \cdot \frac{3}{2} (x + 1) \cdot (3x)$$

$$f'(x) = \sqrt{x^3 + 1} + \frac{3x}{2} (x + 1)^{1/2}.$$ 

The slope at $(2, 6)$ is 

$$m = f'(2) = \sqrt{8 + 1} + \frac{3}{2} \left( \frac{8}{9} \right)^{1/2} - \frac{3 \cdot 4}{2} = \sqrt{9} + \frac{2 \cdot 4}{2} = 7.$$ 

So, $m = 7$

The equation of the tangent line is 

$$y - 6 = 7(x - 2).$$
5. (15 points) Suppose $f(x)$ is differentiable, and satisfies

\[ f'(x) < 0 \quad 0 \quad f'(x) > 0 \quad 0 \quad f'(x) < 0 \quad 0 \quad f'(x) > 0 \quad 0 \quad f'(x) < 0 \]

\[ -3 \quad \frac{1}{2} \quad 1 \quad 2 \quad 5 \]

(a) Find all open intervals where the graph of $f(x)$ is increasing.

\[ (-3, \frac{1}{2}), (1, 2), (2, 5) \]

(b) Find all open intervals where the graph of $f(x)$ is decreasing.

\[ (-\infty, -3), (\frac{1}{2}, 1), (5, \infty) \]

(c) Find the $x$-value for each relative minimum on the graph of $f(x)$.

\[ x = -3 \]
\[ x = 1 \]

(d) Find the $x$-value for each relative maximum on the graph of $f(x)$.

\[ x = \frac{1}{2} \]
\[ x = 5 \]
6. (10 points) A function $f(x)$ is twice differentiable and has the following second derivative
\[ f''(x) = 8(x^2 + 6)(x - 3)^4(x - 5)^7(e^x + 1) \]

(a) Find all open intervals where the function $f(x)$ is concave up.

\[ (5, \infty) \]

(b) Find all open intervals where the function $f(x)$ is concave down.

\[ (-\infty, 3) \text{ and } (3, 5) \]

(c) Find the $x$-coordinate of all inflection points.

\[ x = 5 \]
7. (8 points) You do not need to show work for this question. Just circle the best answer for each part. (2 points each)

(a) Consider the following table of values for \( f, g, f', \) and \( g' \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
<th>( f'(x) )</th>
<th>( g(x) )</th>
<th>( g'(x) )</th>
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If \( p(x) = g(f(x)) \). What is \( p'(1) \)? Circle your answer below.

(A) 20     (B) 30     (C) 36     (D) 40     (E) 50

\[ p'(x) = \frac{g'(f(x)) f'(x)}{f(x)} \]

\[ p'(1) = \frac{g'(f(1)) f'(1)}{f(1)} = \frac{g'(3)}{4} = \frac{(9)(4)}{4} = 36 \]

(b) Suppose that the cost function for a product is given by \( C(q) = 5q^2 - 12q \), where \( q \) is the quantity produced. The marginal average cost of producing 3 units is:

(A) 5     (B) 2     (C) 10     (D) 3     (E) 7

\[ \bar{C}(q) = \frac{5q^2 - 12q}{q} = 5q - 12 \]

\[ \bar{C}'(q) = 5 \]

(c) The derivative of \( f(x) = x^{2x} \) is:

(A) \( x^{2x}(2 \ln x + 1) \)     (B) \( x^2 \ln x + 2 \)     (C) \( 2x^{2x}(\ln x + 1) \)     (D) \( 2x^{2x-1} \)

\[ \ln f(x) = 2x \ln x \]

\[ \frac{d}{dx}(\ln f(x)) = \frac{d}{dx}(2x \ln x) \Rightarrow \frac{1}{f(x)} f'(x) = (2 \ln x + 2x) \frac{1}{x} \Rightarrow f'(x) = f(x) \left(2 \frac{\ln x + 2}{x} x\right) \]

\[ = 2x \left(x^{2x}(\ln x + 1)\right) \]

(d) Let \( f(x) = x^2 + 4x \) and \( g(x) = 3x + 2 \). Circle the function below that represents \( g(f(x)) \).

(A) \( 3x^2 + 12x + 2 \)     (B) \( 7x^2 + 20x + 12 \)     (C) \( 9x^2 + 24x + 12 \)     (D) \( 3x^2 + 4x + 12 \)

\[ g(f(x)) = 3(f(x)) + 2 \]

\[ = 3(x^2 + 4x) + 2 \]

\[ = 3x^2 + 12x + 2 \]
8. (8 points) In 1995, the rate of violent crime in New York City continued to decrease, but at a slower rate than in previous years. Letting \( f(t) \) be the rate of violent crime as a function of time. What does this statement tell you about \( f(t) \), \( f'(t) \), and \( f''(t) \)? Circle your answer below.

(a) \( f(t) \) is: 
- increasing
- decreasing

(b) \( f(t) \) is: 
- concave up
- concave down

(c) \( f''(t) \) is: 
- positive
- negative

(d) \( f'(t) \) is: 
- positive
- negative