Math 595 Calculus on Meshes

Spring 2018 (Half-semester: Second 8 Weeks)

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Course Description: Introduction to finite element discretization of exterior calculus on piecewise linear manifold simplicial complexes (meshes). Approximate syllabus:

1. Introduction
2. Basic homological algebra [1 lecture]
   2.1 Chain complexes (example: simplicial chain complex) and chain maps
   2.2 Cochain complexes (example: de Rham complex)
3. Unbounded operators on Hilbert spaces [2-3 lectures]
   3.1 Unbounded and closed operators
   3.2 Adjoints of unbounded operators
   3.3 Examples: grad, curl, div and their adjoints
4. Hilbert complexes [5-6 lectures]
   4.1 Hilbert complexes and their duals
   4.2 Harmonic forms and Hodge decomposition
   4.3 Poincaré inequality
   4.4 $L^2$ de Rham complex
   4.5 Abstract Hodge Laplacian and the Poisson's equation
   4.6 Formulations of Poisson's equation
   4.7 Well-posedness
   4.8 Hodge Laplacian in $\mathbb{R}^3$
   4.9 Boundary conditions
5. Approximation of Hilbert complexes [3-4 lectures]
   5.1 Galerkin discretization of the primal problem
   5.2 Mixed Galerkin discretization
   5.3 Properties of subspaces
   5.4 Consistency, stability, convergence
6. $L^2$ differential forms [2 lectures]
7. Finite element differential forms [4-5 lectures]
   7.1 Polynomial differential forms $\mathcal{P}_r^- \Lambda^k$ and $\mathcal{P}_r^+ \Lambda^k$
   7.2 Koszul complex
   7.3 Decompositions of $\mathcal{P}_r^- \Lambda^k$ and $\mathcal{P}_r^+ \Lambda^k$
   7.4 Shape functions and degrees of freedom
   7.5 Properties of the finite element spaces
   7.6 Whitney forms ($\mathcal{P}_r^- \Lambda^k$ spaces)