Differential forms, exterior derivative, wedge product and covariant derivative in $\mathbb{R}^3$

1. Practice with 1-forms. 1.5.3

2. Differential as a linear approximation of change in function value. 1.5.11 (a)

3. Practice with exterior derivative and wedge product. 1.6.1, 1.6.4

4. Volume form in cylindrical coordinates. The 3-form $dx \wedge dy \wedge dz$ is called a volume form on $\mathbb{R}^3$. (In vector calculus this is sometimes called the volume element $dx\,dy\,dz$ and used in integrals over volumes.) Compute the expression for this volume form in cylindrical coordinates.

5. Another case of $d^2 = 0$. In class we checked that for $f : \mathbb{R}^3 \to \mathbb{R}$ a differentiable function, $d(df) = 0$. Now let $\phi = f\,dx + g\,dy + h\,dz$ be a 1-form in $\mathbb{R}^3$. Here $f, g, h : \mathbb{R}^3 \to \mathbb{R}$ are all differentiable functions. Show that $d(d\phi) = 0$.

6. Wedge product and determinant. 1.6.5, 1.6.9

7. Practice with covariant derivative. 2.5.2 (a), (d) and (f)

8. Covariant derivative of constant length vector field. 2.5.3

9. Covariant derivative along velocity of a curve. 2.5.5