The theory of knowledge spaces was first introduced by Jean-Claude Falmagne in 1985 in order to build an efficient machine for assessing knowledge.

This theory was fully developed by Jean-Claude Falmagne and Jean-Paul Doignon.

Practical application: ALEKS (Assessment and Learning in Knowledge Spaces)

In 2014, ALEKS had more than 1,900,000 single users.

A knowledge structure is a pair \((Q, K)\) with \(Q \neq \emptyset\) and \(K\) a family of subsets of \(Q\) such that \(Q \in K\) and \(\emptyset \in K\).

\(Q\) is called the domain of the knowledge structure;

Elements of \(Q\) are items or concepts;

Elements of \(K\) are called knowledge states.

Example: Given a test, \(Q\) is the set of all concepts that are tested, and a knowledge state is a set of all concepts mastered by a student.

A knowledge structure \((Q, K)\) is called a learning space if it satisfies the following conditions:

1. Learning smoothness: For any states \(K\) and \(L\) in \(K\) with \(K \subseteq L\), there exists a finite chain of states \(K = K_1 \subseteq K_2 \subseteq \cdots \subseteq K_p = L\) such that \(|K \cap K_i| = 1\) for \(1 \leq i \leq p\) where \(p = |L \setminus K|\).

   Learning is done one step at a time

2. Learning consistency: For any states \(K\) and \(L\) in \(K\) with \(K \subseteq L\), if \(q\) is an item such that \(K \cup \{q\}\) is a state, then \(L \cup \{q\}\) is also a state.

(Naing more does not prevent learning something new).

Learning Spaces

A learning space is a pair \((Q, K)\) such that \(Q \in K\) and \(\emptyset \in K\).

\(Q\) is the set of all concepts that are tested, and \(K\) is the set of all concepts mastered by a student.

\(K\) is an item such that \(K \subseteq Q\).

\(K\) is called a knowledge state.

The diagram on the left represents a learning space developed from 2013 data. We found questions corresponding to various items, and using the performance data, we simulated multiple tests to determine which state is dominant. The diagram on the right represents a learning space that spans several years of data and compares their dominant states.

Progressive Test

We simulate two tests that progress through questions based on student responses, one with increasing difficulty and the other with decreasing difficulty. These special orderings eliminate non-feasible states.

Probability Variations

Uses a normal distribution to calculate a variation in probabilities of correct responses by students at a certain percentile. Identifies most frequent knowledge state for each percentile and finds when a dominant state changes.

Knowledge Spaces:
The Mathematics Behind Assessment and Learning

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