Math 408, Spring 2005
Final Exam
5/9/2005

Name:

Instructions

1. Closed notes, books, etc. You can use a basic calculator, of the type permitted in actuarial exams, but it is needed only for a few very simple calculations. Tables for the normal and chi-square distribution will be distributed.

2. There are 16 problems, worth a total of 90 points. The multipart problems 1 and 10 are worth 10 points each; all others are worth 5 points each.

3. **Show all work and circle the final answer. A correct answer without work or justification will not earn credit.**

4. When you are finished, double-check your work (you have plenty of time!) and cross out everything you do not want to be considered for grading (e.g., a solution you later realized as being incorrect).

5. The exam will be graded by the end of Wednesday, at the latest. You can access your exam score and course grade as usual, via the link on the course webpage.

Problem	1	2/3	4/5	6/7	8/9	10	11/12	13/14	15/16
Points									

1. Assume A and B are independent events with $P(A) = 0.2$ and $P(B) = 0.3$. Let C be the event that **neither** A nor B occurs, let D be the event that **exactly one** of A or B occurs.

   (a) Find $P(C)$.

   (b) Find $P(D)$.

   (c) Find $P(A|D)$.

   (d) Are C and D independent? Justify your answer!

2. Suppose A, B, and C are mutually independent events with probabilities $P(A) = 0.5$, $P(B) = 0.8$, and $P(C) = 0.3$. Find the probability that **at least one** of these events occurs.

3. How many ways are there to seat 10 people, consisting of 5 couples, in a row of seats (10 seats wide) if all couples are to get adjacent seats?

4. The probability that a randomly chosen male has a circulation problem is 0.25. Males who have a circulation problem are twice as likely to be smokers as those who do not have a circulation problem. What is the conditional probability that a male has a circulation problem, given that he is a smoker?

 Show all work and circle/box *your answers*

5. Suppose a random variable X has moment generating function

$$M(t) = \left(\frac{2+e^t}{3}\right)^9.$$

Calculate the variance of X.

6. The lifetime of a printer costing 200 is exponentially distributed with mean 2 years. The manufacturer agrees to pay a full refund to a buyer if the printer fails during the first year following its purchase, and a one-half refund if it fails during the second year. If the manufacturer sells 100 printers, how much should it expect to pay in refunds?

7. An insurance policy is written to cover a loss, X, where X has uniform distribution on $[0, 1000]$. At what level must a deductible be set in order for the expected payment to be 25% of what it would be with no deductible?

8. A device that continuously measures and records seismic activity is placed in a remote region. The time, T, to failure of this device is exponentially distributed with mean 3 years. Since the device will not be monitored during its first two years of service, the time to discovery of its failure is $X = \max(T, 2)$. Determine $E(X)$.

9. Claim amounts for wind damage to insured homes are independent random variables with common density function $f(x) = 3x^{-4}$ for $x > 1$, and $f(x) = 0$ otherwise, where x is the amount of a claim in thousands. Suppose 3 such claims are made. What is the expected value of the largest of the three claims?

10. Let X have uniform distribution on the interval $[0, 2]$, and given $X = x$, let Y have uniform distribution on the interval $[0, x^2]$.

    (a) Find the **joint density** $f(x, y)$ of X and Y. (Be sure to specify the range!)

    (b) Find the **marginal density** $f_Y(y)$ of Y. (Be sure to specify the range!)

    (c) Find $E(XY)$.

11. Let X and Y be random variables with joint density

$$f(x, y) = x - y + 1, \quad 0 \leq x, y \leq 1.$$

Find the probability that $X + Y \geq 0.5$.

12. A computer generates 48 random real numbers, rounds each number to the nearest integer and then computes the average of these 48 rounded values. Assume that the numbers generated are independent of each other and that the rounding errors are distributed uniformly on the interval $[-0.5, 0.5]$. Find the approximate probability that the average of the rounded values is within 0.05 of the average of the exact numbers.

13. A company manufactures a brand of light bulb with a lifetime in months that is normally distributed with mean 3 and variance 1. A consumer buys a number of these bulbs with the intention of replacing them successively as they burn out. The light bulbs have independent lifetimes. What is the smallest number of bulbs to be purchased so that the succession of light bulbs produces light for at least 40 months with probability 0.9772?

14. Assume the math scores on the SAT test are normally distributed with mean 500 and standard deviation 60, and the verbal scores are normally distributed with mean 450 and standard deviation 80. If two students who took both tests are chosen at random, what is the probability that the first student's math score exceeds the second student's verbal score?

15. Let X_1, X_2, X_3, X_4 be a random sample of size 4 from the normal distribution $N(76.4, 383)$, and let $\overline{X}$ be the sample mean and S^2 the sample variance. Determine a such that $P(S^2 \leq a) = 0.90$.

16. Suppose X and Y are independent, each having Poisson distribution with means 2 and 3, respectively. Let $Z = X + Y$. Find $P(X + Y = 1)$.