Setting $X = x + \frac{1}{x}$, we have

$$X^3 + C_n X^2 - (C_n - 6)X - (C_n^2 + 2C_n - 16) = 0,$$

and with $X = Y - C_n/3$, we obtain

$$Y^3 + (6 - C_n - C_n^2/3)Y + (16 - 4C_n - 2C_n^2/3 + 2C_n^3/27) = 0.$$

Put $Y = u + v$, $p = 6 - C_n - C_n^2/3$ and $q = 16 - 4C_n - 2C_n^2/3 + 2C_n^3/27$. Thus u^3 and v^3 are solutions of $Z^2 + qZ - p^3/27 = 0$, and so

$$u = \sqrt[3]{-27q/2 + 3\sqrt{3\Delta}/2},\ a_1 = \frac{u + \overline{u} - C_n}{3},\ a_2 = \frac{ju + \overline{ju} - C_n}{3},\ a_3 = \frac{j^2 u + \overline{j^2 u} - C_n}{3},$$

where $\overline{u}$ is the complex conjugate of u. Therefore

$$\nu_1 = \frac{a_2 - \sqrt{a_2^2 - 4}}{2},\quad \nu_2 = \frac{a_1 + \sqrt{a_1^2 - 4}}{2},\quad \nu_3 = \frac{a_3 + \sqrt{a_3^2 - 4}}{2},$$

$$\nu_4 = \frac{a_2 + \sqrt{a_2^2 - 4}}{2},\quad \nu_5 = \frac{a_1 - \sqrt{a_1^2 - 4}}{2},\quad \nu_6 = \frac{a_3 - \sqrt{a_3^2 - 4}}{2},$$

whereby $\varepsilon = |\nu_1|$, $\varepsilon' = |\nu_2|$.

We determine the constants X_0, X_1, X_2, X_3, $c_1, \ldots, c_{12}$, δ_i and λ_i:

$$X_0 = 1,\ c_1 = \frac{16\theta(\theta - 1)(\theta + 1)(\theta + 2)(2\theta + 1)^5}{9(\theta^2 + \theta + 1)^5},\ c_2 = \frac{\theta^2 + \theta + 1}{(2\theta + 1)(\theta + 1)},$$

$$X_1 = \max(X_0, (7c_1/c_2)^{1/6}),\ c_3 = \frac{7}{6(\theta^2 + \theta + 1)} c_1,\ c_4 = 1.39 c_3, X_2 = \max(X_1, (2c_3)^{1/6}),$$

$$c_5 = \begin{cases} \left|\frac{\log(\varepsilon_3^2 \varepsilon_3')}{U}\right| & if\ n = 0, \\ \left|\frac{\log(\varepsilon^2/\varepsilon')}{V}\right| & \text{otherwise}, \end{cases}$$

$$c_6 = \begin{cases} -\frac{\log(\varepsilon_3^2 \varepsilon_3')}{6U} + \left|\frac{-\log(\varepsilon_3)\log(T_2 T_5) - \log(\varepsilon_3 \varepsilon_3')\log(T_3 T_6)}{2U}\right| & \text{if } n = 0, \\ \frac{\log(\varepsilon^2/\varepsilon')}{6V} + \left|\frac{\log((\varepsilon')^2/\varepsilon)\log(T_2) + 3\log(\varepsilon'/\varepsilon)\log(T_3) + 2\log(\varepsilon'/\varepsilon^2)\log(T_4) - 3\log(\varepsilon)\log(T_5) - \log(\varepsilon\varepsilon')\log(T_6)}{6V}\right|, \end{cases}$$

where

$$T_j = \theta - \theta_j,\quad j \neq 1,\quad U = \log^2(\varepsilon_3) + \log^2(\varepsilon_3') + \log(\varepsilon_3)\log(\varepsilon_3'),$$

$$V = \log^2(\varepsilon) + \log^2(\varepsilon') - \log(\varepsilon)\log(\varepsilon'),$$

$$c_7 = c_5,\quad c_8 = c_6,\quad c_9 = c_4 \exp(6c_6/c_5),\quad c_{10} = 6/c_5$$

because $\mathbb{K}_t$ is real;

$$c_{11} = 7 \times 5 \times 3^{14} \times 2^{59} \log(72) \log(\varepsilon_2) \log^2(\varepsilon_3^{-2}(\varepsilon_3')^{-1}) \log^2(\varepsilon) \log[9(\theta + 2)],$$

$$B_0 = \max\left(e, 2c_{10}^{-1} c_{11} \log\left(c_{10}^{-1} c_{11} c_9^{1/c_{11}}\right)\right),$$

$$i_1 = \begin{cases} 2 & if\ n = 0, \\ 4 & \text{otherwise}, \end{cases} \qquad i_2 = 1,$$

$$\delta = \begin{cases} \delta_1/\delta_2 & if\ n = 0, \\ \delta_1/\delta_4 & \text{otherwise}, \end{cases} \qquad \lambda = \begin{cases} (\delta_1\lambda_2 - \delta_2\lambda_1)/\delta_2 & if\ n = 0, \\ (\delta_1\lambda_4 - \delta_4\lambda_1)/\delta_4 & \text{otherwise}, \end{cases}$$

$$c_{14} = c_{12}\exp(6c_6/c_5), c_{15} = 6/c_5,$$

$$\begin{cases} \delta_1 = -\frac{1}{\log(\varepsilon_2)}, \qquad \delta_2 = -\frac{\log(\varepsilon_3^2\varepsilon_3')}{U}, \quad \delta_3 = -\frac{\log(\varepsilon_3(\varepsilon_3')^2)}{U}, \\ \delta_4 = -\frac{\log(\varepsilon^2/\varepsilon')}{V}, \quad \delta_5 = \frac{\log(\varepsilon/(\varepsilon')^2)}{V}, \end{cases}$$

and

$$\begin{cases} \lambda_1 = -\frac{\log(T_2T_4T_6)}{3\log(\varepsilon_2)}, \\ \lambda_2 = \frac{-\log(\varepsilon_3)\log(T_2T_5)-\log(\varepsilon_3\varepsilon_3')\log(T_3T_6)}{2U}, \\ \lambda_3 = \frac{-\log(\varepsilon_3\varepsilon_3')\log(T_2T_5)-\log(\varepsilon_3')\log(T_3T_6)}{2U}, \\ \lambda_4 = \frac{\log((\varepsilon')^2/\varepsilon)\log(T_2)+3\log(\varepsilon'/\varepsilon)\log(T_3)+2\log(\varepsilon'/\varepsilon^2)\log(T_4)-3\log(\varepsilon)\log(T_5)-\log(\varepsilon\varepsilon')\log(T_6)}{6V}, \\ \lambda_5 = \frac{-\log(\varepsilon\varepsilon')\log(T_2)-3\log(\varepsilon')\log(T_3)-2\log((\varepsilon')^2/\varepsilon)\log(T_4)-3\log(\varepsilon'/\varepsilon)\log(T_5)-\log(\varepsilon'/\varepsilon^2)\log(T_6)}{6V}. \end{cases}$$

Using the following system

$$\begin{cases} \gamma_1 = x - \theta y = \varepsilon_2^{b_1}\varepsilon_3^{b_2}(\varepsilon_3')^{b_3}\varepsilon^{b_4}(\varepsilon')^{b_5}, \\ \gamma_2 = x - \theta_2 y = \varepsilon_2^{-b_1}(\varepsilon_3')^{b_2}\left(\varepsilon_3\varepsilon_3'\right)^{-b_3}(\varepsilon')^{b_4}\left(-\varepsilon^{-1}\varepsilon'\right)^{b_5}, \end{cases}$$

we obtain

$$\begin{cases} x = \frac{-(\theta-1)\gamma_1+\theta(\theta+2)\gamma_2}{\theta^2+\theta+1}, \\ y = \frac{(\theta+2)(-\gamma_1+\gamma_2)}{\theta^2+\theta+1}. \end{cases}$$

The computations are done with a SUN PARC ULTRA1, and for each value of n, the time of computation is roughly 9 seconds.