Mini-Course: Introduction to Random Walks

Tuesdays/Thursdays, 2:00 pm - 3:30 pm, 345 Altgeld Hall
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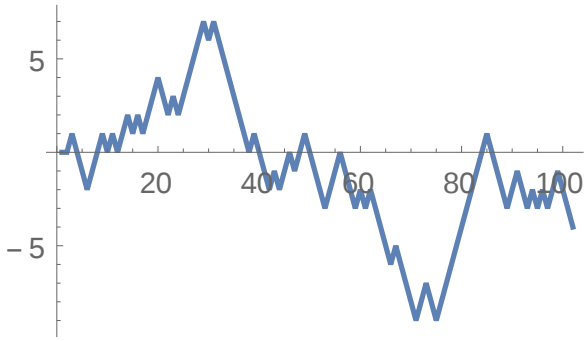
About this Course

Random walks arise in a wide variety of real-life situations: The ups and downs of a stock market, the progression of scores in a basketball game, and the tallying of votes in an election, can all be modeled by appropriate random walks. This course provides an elementary introduction to this topic, accessible to anyone who has taken a basic undergraduate course in probability or statistics.



Tentative List of Topics

- **Probability of Return.** What is the probability that the random walk eventually returns to its starting position? This question, and its generalization to higher-dimensional random walks, is known as the "Drunkard's Walk Problem."

- **The Gambler's Ruin Problem.** If you repeatedly play a fair game, what is the probability that you eventually lose your starting capital? What is the probability that you eventually win one million dollars? Surprisingly, both of these events have probability 1, i.e., they are "guaranteed" to occur. This is a special case of the "Gambler's Ruin Problem."

- **Lead Changes, Times in Lead, and the Ballot Problem.** How many "lead changes" (or "ties") should we expect in a basketball game with scores occurring according to a random walk? What can we say about the amount of time a given team is in the lead? How likely it that the winning team has been in the lead throughout the game? What is the probability that the winner in a two candidate election has been in the lead through the entire vote counting process? (The latter question is the "Ballot Problem.")

  All of these questions can be analyzed precisely. The mathematics behind this analysis involves one of the most ingenious "tricks" I know of in all of mathematics—the so-called "reflection principle"—and it leads to an unusual probability distribution, the arcsine distribution.

- **Probability of Streaks.** A "streak" is a sequence of consecutive losses or consecutive wins. The world of sports is filled with famous examples of streaks of various kinds. For example, a baseball team that has won its last ten games is said to have had a 10-game win streak. How likely is such a streak in a 162 game season assuming wins and losses occur randomly? What is the expected length of the longest streak in a 162 game season?

  Questions of this type can be answered mathematically rather precisely. Most of the famous sports streaks turn out to match the mathematical model fairly closely.

- **Maximizing Investment Returns.** One of the most famous investment/gambling strategies is given by "Kelly's Formula," a strategy that, in an appropriate sense, maximizes the long-term growth of the investment. It was discovered some fifty years ago, it has been tested in casino gambling, and it is widely used in the investment industry. The formula has a fascinating history that is the subject of a book titled "Fortune's Formula." (The title refers to Kelly's Formula.) Mathematically, Kelly's formula is based on a simple random walk model and can be derived in this context.

- **Additional Topics.** Depending on time available and interests and backgrounds of the participants, we may cover more advanced topics such as Brownian Motion and the famous Black-Scholes formula for the pricing of stock market options.

General information

- **Sign-up.** This course is part of the NU-UI Summer REU Program, but it is open, and free of charge, to other UI students as long as there is space available. To sign up, use the form at `http://go.illinois.edu/mathreu`.

- **Prerequisites.** The main prerequisites are a basic undergraduate probability or statistics course such as Math 461 or Math 463/Stat 400. You should be familiar with basic concepts in probability such as random variables and expectation, and with the standard probability distributions such as the binomial, geometric, and normal distributions.

- **Text/resources, and homework.** There is no text for this course. I plan to distribute summary lecture notes, and I will provide references and links to additional resources where available. There will be homework assignments related to the topics covered.

- **Qestions/suggestions.** If you have any questions or suggestions, you can contact me at `ajh@illinois.edu`. You can also talk to me after class (I will be available after each class through at least 4 pm), or come to my office, 13 Illini Hall.