Problem 1, UI Freshman Math Contest 2012. Determine, with proof, whether there exists a power of 2 whose decimal representation ends in the digits 2012.

Problem 1, UI Freshman Math Contest 2011. Let $x$ be the number whose decimal expansion consists of the sequence of natural numbers written next to each other, i.e., $x = 0.1234567891011121314\ldots$.

(a) Determine the 2011th digit after the decimal point of $x$.
(b) Prove that $x$ is irrational.

Problem A1, Putnam 2012. Let $d_1, d_2, \ldots, d_{12}$ be 12 real numbers in the open interval $(1, 12)$. Show that there exist distinct indices $i, j, k$ such that $d_i, d_j, d_k$ are the side lengths of an acute triangle.

Problem 5, UI Mock Putnam Exam 2008. Let $a_1, a_2, \ldots, a_{65}$ be 65 positive integers, none of which has a prime factor greater than 13. Prove that, for some $i, j$ with $i \neq j$, the product $a_i a_j$ is a perfect square.

Problem 6, UI Undergraduate Math Contest 2012. Call a positive integer defective if its decimal representation does not contain all ten digits 0, 1, 2, $\ldots$, 9. Thus, for example, the number 3141592653589 is defective (since it does not contain the digits 7 and 0), but the number 3141592653589793238462433832795028 is not defective (since it contains each of the digits 0, 1, $\ldots$, 9). Let $D$ denote the set of defective numbers. Determine, with proof, whether the sum of reciprocals of the numbers in $D$ converges or diverges.