Math Contest Sampler

Below are some problems from recent Putnam exams and local math contests, arranged roughly in increasing order of difficulty. For solutions to these problems, and a large collection of additional contest problems, visit the UI Math Contest Website:

`http://www.math.illinois.edu/contests.html`

Problem 1, UI Freshman Math Contest 2012. Determine, with proof, whether there exists a power of 2 whose decimal representation ends in the digits 2012.

Problem 1, UI Freshman Math Contest 2011. Let x be the number whose decimal expansion consists of the sequence of natural numbers written next to each other, i.e., $x = 0.12345678910111213\ldots$.

(a) Determine the 2011th digit after the decimal point of x.

(b) Prove that x is irrational.

Problem A1, Putnam 2012. Let $d_1, d_2, \ldots, d_{12}$ be 12 real numbers in the open interval $(1, 12)$. Show that there exist distinct indices i, j, k such that d_i, d_j, d_k are the side lengths of an acute triangle.

Problem 5, UI Mock Putnam Exam 2008. Let $a_1, a_2, \ldots, a_{65}$ be 65 positive integers, none of which has a prime factor greater than 13. Prove that, for some i, j with $i \neq j$, the product $a_i a_j$ is a perfect square.

Problem 6, UI Undergraduate Math Contest 2012. Call a positive integer defective if its decimal representation does not contain all ten digits $0, 1, 2, \ldots, 9$. Thus, for example, the number 3141592653589 is defective (since it does not contain the digits 7 and 0), but the number 31415926535897932384626433832795028 is not defective (since it contains each of the digits $0, 1, \ldots, 9$). Let D denote the set of defective numbers. Determine, with proof, whether the sum of reciprocals of the numbers in D converges or diverges.