1. Let
\[ f(n) = \sum_{k=1}^{n} \frac{1}{\sqrt{k} + \sqrt{k+1}}. \]
Evaluate \( f(9999) \).

2. Evaluate the integral
\[ \int_{0}^{\infty} \frac{\ln x}{x^2 + 9} \, dx. \]

3. Show that a positive integer whose decimal representation contains each of the digits 1, 2, 3, 4, 5, 6, 7 exactly 3 times and does not contain the digit 8 (but with no restrictions on the number of the digits 0 and 9) cannot be a perfect square.

4. (Virginia Tech Math Contest 2007) Let \( n \) be a positive integer, let \( A, B \) be symmetric \( n \times n \) matrices with real entries. Suppose there exist \( n \times n \) matrices \( X, Y \) such that \( \det(AX + BY) \neq 0 \). Prove that \( \det(A^2 + B^2) \neq 0 \).

5. (B3, Putnam 2001) For any real number \( t \), let \( \langle t \rangle \) denote the integer closest to \( t \); for example, \( \langle 3.14159 \rangle = 3 \) and \( \langle 2.71828 \rangle = 3 \). Evaluate
\[ \sum_{n=1}^{\infty} \frac{2^{\langle \sqrt{n} \rangle} + 2^{-\langle \sqrt{n} \rangle}}{2^n}. \]

6. Let \( a_1, a_2, a_3, \ldots \) be positive real numbers, and let \( S_n = \sum_{k=1}^{n} a_k \).

(a) (Virginia Tech Math Contest, 1979) Prove or disprove: If the series \( \sum_{n=1}^{\infty} a_n \) diverges, then the series \( \sum_{n=1}^{\infty} \frac{a_n}{S_n^2} \) converges.

(b) (A5, Putnam 1964, variation) Prove or disprove: If the series \( \sum_{n=1}^{\infty} \frac{1}{a_n} \) converges, then the series \( \sum_{n=1}^{\infty} \frac{n}{S_n} \) converges as well.

[Solutions at http://www.math.uiuc.edu/contests.html]