1. Prove that, given any power of 2 (such as 1024), there exist infinitely many powers of 2 whose decimal representation ends with the digits of the given power of 2.

2. Determine, with proof, all positive integers \( n \) for which there is a polynomial of degree \( n \) satisfying the following three conditions:
   (i) \( P(k) = k \) for \( k = 1, 2, \ldots, n \);
   (ii) \( P(0) \) is an integer;
   (iii) \( P(-1) = 2012 \).

3. Given positive integers \( n \) and \( m \) with \( n \geq 2m \), let \( f(n, m) \) be the number of binary sequences of length \( n \) (i.e., strings \( a_1a_2\ldots a_n \) with each \( a_i \) either 0 or 1) that contain the block 01 exactly \( m \) times. For example, the sequence 10(01)11(01)0(01) contains this block 3 times.

   Find, with proof, a simple formula for \( f(n, m) \).

4. Let \( x_0 = 0, \ x_1 = 1, \) and
\[
x_{n+1} = \frac{1}{n+1} x_n + \left( 1 - \frac{1}{n+1} \right) x_{n-1} \quad (n \geq 1).
\]
Show that the sequence \( \{x_n\} \) converges as \( n \to \infty \) and determine its limit.

5. [A4, Putnam 1998] Let \( A_1 = 0, \ A_2 = 1, \) and for \( n > 2 \) define \( A_n \) as the number obtained by concatenating the numbers \( A_{n-1} \) and \( A_{n-2} \) (written in decimal). Thus, \( A_3 = A_2A_1 = \boxed{10} = 10, \ A_4 = A_3A_2 = \boxed{101} = 101, \ A_5 = A_4A_3 = \boxed{10110} = 10110, \) and so on.

   Determine, with proof, the set of \( n \) for which \( A_n \) is divisible by 11.

6. [B4, Putnam 1988] Let \( a_1, a_2, a_3, \ldots \) be a sequence of positive real numbers, and let \( A_n = \sqrt[n]{a_n} \). Prove that if the series \( \sum_{n=1}^{\infty} \frac{1}{a_n} \) converges, then so does the series \( \sum_{n=1}^{\infty} \frac{A_n}{a_n} \).

[Solutions at http://www.math.uiuc.edu/contests.html]