UIUC Mock Putnam Exam 1/2007

(Solutions at `www.math.uiuc.edu/contests.html`)

Problem 1. Given an integer $n \geq 2$, let $f(n)$ denote the number of ordered pairs of non-empty, disjoint subsets of an n-element set. Find a simple formula for $f(n)$.

Problem 2. Without any numerical calculations, determine which of the two numbers 3.14^{π} and $\pi^{3.14}$ is larger.

Problem 3. Let $a_0 = 1$, $a_1 = 1$, $a_2 = 2$, and for $n \geq 3$ define a_n to be the last digit of the sum of the preceding three terms in the sequence. Thus the first few terms of this sequence of digits are (in concatenated form) 1124734419447... Determine whether or not the string 1001 occurs somewhere in this sequence.

Problem 4. How many 8 by 8 matrices are there in which each entry is 0 or 1 and each row and each column contains an odd number of 1's? Explain!

Problem 5. Determine, with proof, whether the series

$$\sum_{n=1}^{\infty} \frac{1}{n^{1.7+\sin n}}$$

converges or diverges.