Problem 1. Evaluate the integral

\[ I = \int_0^\pi \ln(\sin x) \, dx. \]

Problem 2. Let \( a, b, c \) denote distinct integers. Does there exist a polynomial with integral coefficients that cyclically permutes these values in the sense that \( P(a) = b, P(b) = c \) and \( P(c) = a \)? Explain!

Problem 3. [Putnam 1966, A3] Let \( 0 < x_0 < 1 \), and \( x_{n+1} = x_n(1 - x_n) \) for \( n \geq 0 \). Prove that the limit \( \lim_{n \to \infty} nx_n \) exists and is equal to 1.

Problem 4. Evaluate \( \sum_{k=0}^n \binom{n}{k}^2 (-1)^k. \)