Problem 1. How many positive integers are there whose binary expansion has exactly 2003 digits and the sum of its binary digits is even.

Problem 2. Can 2003 be expressed as a sum of the squares of two integers?

Problem 3. If $n$ is a positive integer, count the solutions in nonnegative integers $x_1, \ldots, x_k$ to the equation $x_1 + \cdots + x_k \leq n$.

Problem 4. In how many ways can one distribute 100 slices of pizza between 20 students if every student gets at least one slice?

Problem 5. Evaluate the integral

$$I = \int_0^{\pi/2} \frac{dx}{1 + (\tan x)^{\sqrt{2}}}.$$