1. Find a function \( f(x) \) such that \( f(f(x)) = 2018x + 2017 \) for all real numbers \( x \) or show that no such function exists.

2. Consider the equation

\[
a_1a_2 + a_2a_3 + \cdots + a_{n-1}a_n + a_na_1 = 0,
\]

where each \( a_i \) is either +1 or −1.

(a) Find a sequence \( a_i = \pm 1, \ i = 1, \ldots, 2016 \), that satisfies (1) for \( n = 2016 \).

(b) Prove that, when \( n = 2018 \), (1) has no solution in integers \( a_i = \pm 1 \).

3. Let \( s(n) \) denote the sum of the decimal digits of an integer \( n \). For example, \( s(2018) = 2 + 0 + 1 + 8 = 11 \).

Find the smallest positive integer \( n \) such that \( s(n) \) and \( s(n + 1) \) are both divisible by 17 or show that no such integer exists.

4. Determine the set of nonnegative integers that can be written as \( \lfloor x \rfloor + \lfloor 2x \rfloor + \lfloor 4x \rfloor \) for some positive real number \( x \). (\( \lfloor x \rfloor \) is the floor function, defined as the largest integer \( \leq x \).)

5. Given a finite nonempty set \( A \) of real numbers, let \( P(A) \) denote the product of all elements of \( A \). For example, \( P\{2, 5, 6\} = 2 \cdot 5 \cdot 6 = 60 \). Set \( P(\emptyset) = 1 \) (\( \emptyset \) denotes the empty set). Evaluate, with proof, the sum

\[
\sum_{A \subseteq \{1, 2, \ldots, 2018\}} \frac{1}{P(A)},
\]

where \( A \) runs over all subsets of the set \( \{1, 2, \ldots, 2018\} \) (including the empty subset).

6. Define a sequence \( a_1 < a_2 < a_3 < \cdots \) of positive integers as follows. Let \( a_1 = 1 \) and for each \( n \geq 2 \) let \( a_n \) be the smallest integer greater than \( a_{n-1} \) that is not of the form \( 2a_k \) for some \( k < n \). The first few terms of this sequence are 1, 3, 4, 5, 7, 9, 11, 12, 13, 15, 16, 17, 19, 20, \ldots.

How many terms of this sequence are \( \leq 2048 \)? Justify your answer.