1. Let \( \{a_n\} \) be the sequence defined by \( a_0 = 0, a_1 = 1, a_2 = 2, \) and
\[
a_n = a_{n-1} + a_{n-2} - a_{n-3} + 1
\]
for \( n \geq 3 \). Find, with proof, \( a_{2013} \).

2. (Problem A1, Putnam 1993) The horizontal line \( y = c \) intersects the curve \( y = 2x - 3x^3 \) in the first quadrant as in the figure. Find, with proof, \( c \) so that the areas of the two shaded regions are equal.

3. Prove that the product of any 2013 consecutive integers (for example, \( 1000 \cdot 1001 \cdots 3010 \cdot 3012 \)) is divisible by 2013! (i.e., by \( 1 \cdot 2 \cdots 2012 \cdot 2013 \)).

4. Let \( f(n) \) be the number of \( n \)-letter words that can formed with the letters A,B,C,D and such that the letter A occurs an even number of times. For example, when \( n = 1 \), there are 3 such words, namely B,C,D, so \( f(1) = 3 \); when \( n = 2 \), there are 10 such words, namely AA,BB,BC,BD,CC,CD,DB,DC,DD, so \( f(2) = 10 \). Find, with proof, a simple formula for \( f(n) \). (The formula should not involve a summation of more than two terms.)

5. Show that the number
\[
N = 1^{2013} + 2^{2013} + \cdots + 2013^{2013}
\]
is divisible by \( 2013^2 \).

6. Show that the polynomial
\[
P_n(x) = 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^{2n}}{(2n)!}
\]
(where \( n \) is a natural number) has no real roots.

[Solutions at \texttt{http://www.math.uiuc.edu/contests.html}]