1. For which positive integers \( n \) does the equation
\[
a_1a_2 + a_2a_3 + \cdots + a_{n-1}a_n + a_na_1 = 0
\]
have a solution in integers \( a_i = \pm 1 \)? Explain!

2. Evaluate the integral \( I = \int_{0}^{\pi} \ln(\sin x) \, dx \).

3. Suppose 3 players, \( P_1, P_2, P_3 \), seated at a round table, take turns rolling a die. Player \( P_1 \) rolls first, followed by \( P_2 \), etc. Once a player has rolled a 6, the game is stopped and that player is declared the winner. If no 6 has been obtained after each of \( P_1, P_2, P_3 \) have rolled the die once, player \( P_1 \) gets to roll again, followed by \( P_2 \), etc. Find the probability that the first player, \( P_1 \), wins this game.

4. Prove that, for any real numbers \( x \) and \( y \) in the interval \((0, 1)\),
\[
\left( \frac{x + y}{2} \right)^{x+y} \leq x^x y^y.
\]

5. Determine, with proof, whether the series
\[
\sum_{n=1}^{\infty} \frac{1}{n \cdot 1.8 + \sin n}
\]
converges or diverges.

6. Let \( a_1, \ldots, a_n \) be a set of positive integers such that the product \( \prod_{i=1}^{n} a_i \) has fewer than \( n \) distinct prime divisors. Prove that there exists a nonempty subset \( I \subset \{1, \ldots, n\} \) such that \( \prod_{i \in I} a_i \) is a perfect square.