Problem 1
Suppose that \( a_1, a_2, \ldots, a_n \) are \( n \) given integers. Prove that there exist integers \( r \) and \( s \) with \( 0 \leq r < s \leq n \) such that \( a_{r+1} + a_{r+2} + \ldots + a_s \) is divisible by \( n \).

Problem 2
Let \( p \) be a point inside a triangle having sides of lengths \( a, b, c \). Let \( h_a \) be the distance from \( p \) to the side of length \( a \), and let \( h_b \) and \( h_c \) be defined analogously. Let \( h := \min(h_a, h_b, h_c) \) and \( s := (a + b + c)/2 \). Prove that
\[
h \leq \sqrt{(s-a)(s-b)(s-c)/s}.
\]

Problem 3
Let \( f \) and \( g \) be twice continuously differentiable functions on \([0, 1]\) with \( f(0) = g(0) = 0 = f(1) = g(1) \). Suppose that \( 0 < f(x) < g(x) \) for \( 0 < x < 1 \) and that \( f''(x) < 0 \) for \( 0 < x < 1 \). Show that
\[
\int_0^1 f'(x)^2 \, dx \leq \int_0^1 g'(x)^2 \, dx.
\]

Problem 4
Prove that if \( a, b, \) and \( c \) are odd positive integers, then the polynomial \( ax^2 + bx + c \) has no rational roots.

Problem 5
Evaluate the infinite series
\[
\frac{1}{2^1 - 2^{-1}} + \frac{1}{2^2 - 2^{-2}} + \frac{1}{2^4 - 2^{-4}} + \frac{1}{2^8 - 2^{-8}} + \cdots
\]

Problem 6
Let \( f(n) \) denote the number of 1’s in the binary expansion of \( n \). Evaluate the sum
\[
\sum_{n=1}^{\infty} \frac{f(n)}{n(n+1)}.
\]