Continuing with the “Constants” theme, consider real numbers represented by their decimal expansion, such as

\[ \pi = 3.1415926535897932384626433832795028841971693993751 \ldots \]

Given two such numbers, say \( x \) and \( y \), call \( x \) a daughter of \( y \) if the decimal expansion of \( x \) can be obtained by deleting certain (possibly infinitely many) digits of the decimal expansion of \( y \). For example, the number

\[ 0.1196387334243375281763971 \ldots \]

is a daughter of \( \pi \), since it can be obtained by deleting the digit before the decimal point and all digits in positions 2, 4, 6, \ldots after the decimal point in the expansion of \( \pi \).

Does there exist a “mother of all constants”, in the sense that every real number in the interval \((0,1)\) is its daughter?
Solution to “A mother of all constants”

This is an example of a problem where too much knowledge might be harmful. A well-known method to show the existence or nonexistence of numbers of various special types is the Cantor diagonalization method. so it is tempting to try something similar here.

However, the problem is a lot easier if approached from scratch and looked at the right way. The only thing one needs to realize is that in order for a constant to be a “mother of all constants” it is necessary and sufficient that its decimal expansion contains each of the digits \{0, 1, \ldots, 9\} infinitely often. While for most familiar constants (e.g., \(\pi\)) it is not known whether this is the case, it is easy to “manufacture” constants that have this property. One example is the rational number

\[
0.0123456789 = \frac{123456789}{10^{10} - 1}
\]