

UIUC Department of Mathematics

PROBLEM OF THE WEEK

September 28, 2007

Determining a polynomial on the cheap

It is well-known that a polynomial of degree d is completely determined by its values at $d + 1$ distinct points. Moreover, the number $d + 1$ here is best-possible—knowing only d values is not sufficient to determine the polynomial. It may come as a surprise therefore that, if one restricts to polynomials with nonnegative integer coefficients, then the knowledge of only two cleverly chosen values, both at integers, is enough to uniquely determine the polynomial, regardless of its degree. This is the content of this week's POW:

Suppose $P(x)$ is an unknown polynomial, of unknown degree, with nonnegative integer coefficients. Your goal is to determine this polynomial. You have access to an oracle that, given an integer n , spits out $P(n)$, the value of the polynomial at n . However, the oracle charges a fee for each such computation, so you want to minimize the number of computations you ask the oracle to do. Show that it is possible to uniquely determine the polynomial after only two consultations of the oracle.

PROBLEM OF THE WEEK ARCHIVE

<http://www.math.uiuc.edu/~hildebr/pow>