Abstracts of Plenary Talks

Krishnaswami Alladi (University of Florida)

Parity of the generalized divisor function with restrictions on the prime factors

We will discuss the asymptotic behavior of the sum $S_{-k}(x, y) = \sum_{n \leq x}(-k)^{\nu_y(n)}$, for $y \leq x$, where k is a positive integer and $\nu_y(n)$ is the number of prime factors of n less than y. As y falls away from x, the asymptotic behavior is very interesting. There is a difference in behavior when $k = p-1$, where p is prime, and when $k \neq p-1$. In the case $k = p-1$, although the above sum behaves like $xm_k(\alpha)/(\log y)^{k+2}$, where $\alpha = \log x/\log y$, there are terms $x/\log y, x/\log^2 y, \ldots, x/\log^{k+1} y$, but these are accompanied by factors which rapidly go to zero, and so only $x/\log^{k+2} y$ survives. The analysis of the asymptotic behavior of the above sum is done using sieve techniques, analytic methods, and by the study of difference differential equations. This is joint work with my recent PhD student Ankush Goswami.

George Andrews (Pennsylvania State University)

George Beck's conjectures on ranks of partitions

Dyson conjectured (and Atkin and Swinerton-Dyer proved) combinatorial explanations for the Ramanujan congruences for $p(5n + 4)$ and $p(7n + 5)$. George Beck has provided related conjectures where now the total number of parts in the partition is counted. We shall describe the proofs of these conjectures.

Alexander Berkovich (University of Florida)

On a q-theoretic approach to Capparelli's partition theorems

I show how to use the q-binomial theorem to prove two new polynomial identities involving q-trinomial coefficients. Then I apply a trinomial analogue of Bailey's Lemma to convert these identities into another set of two identities, which imply Capparelli's partition theorems. If time permits I will discuss a new infinite hierarchy containing q-series identities for Capparelli's partition theorems.

This talk is based on my recent joint work with Ali K. Uncu.

Bruce Berndt (University of Illinois)

Ramanujan—The Ultimate Superhero (public lecture)

Srinivasa Ramanujan is perhaps the most enigmatic mathematician in the history of our subject. First, a short account of Ramanujan's life will be given. Second, the speaker will provide a history of his association with Ramanujan beginning with Robert Rankin in 1966, Paul Bateman in 1967, Emil Grosswald in 1970, and a cold office at the Institute for Advanced Study in February, 1974. Third, some examples from Ramanujan's Notebooks and Lost Notebook will be given to provide evidence that this great Indian mathematician is, indeed, **The Ultimate Superhero**. This lecture will be aimed at a general audience.

Heng Huat Chan (National University of Singapore)

Ramanujan and the partition function $p(n)$

In Chapter 19 of the classic book "An Introduction to the theory of numbers", G.H. Hardy and E.M. Wright discussed Ramanujan's congruences for $p(n)$, which state that for any nonnegative integers n,

$$p(5n+4) \equiv 0 \pmod 5, \; p(7n+5) \equiv 0 \pmod 7 \text{ and } p(11n+6) \equiv 0 \pmod{11}.$$

The authors then provided Ramanujan's proof for the first congruence and sketched the proof of the second. They then remarked that the proof of the third congruence "is more difficult". I will begin my talk with Ramanujan's "difficult proof" for $p(11n+6) \equiv 0 \pmod{11}$ and his attempt to derive similar results for $p(\ell n - d_\ell)$ where $0 < d_\ell < \ell$ satisfies $d_\ell \equiv (\ell^2-1)/24 \pmod \ell$. An example of Ramanujan's result for $\ell = 17$ is

$$\prod_{k=1}^{\infty}(1-x^{17k})\sum_{n=0}^{\infty}p(17n+5)x^n \equiv 7Q\Delta \pmod{17},$$

where

$$Q = 1 + 240\sum_{j=1}^{\infty}\frac{j^3x^j}{1-x^j}$$

and

$$\Delta = x\prod_{k=1}^{\infty}(1-x^k)^{24}.$$

Song Heng Chan (Nanyang Technological University)

Ranks of partitions and some identities related to mock theta functions

We will give a brief introduction on the ranks of partitions and identities relating ranks of partitions to mock theta functions. We will share some of the results in these two topics including those arising from joint work with Professor Bruce Berndt.

Youn-Seo Choi (Korea Institute for Advanced Study)

Hypergeometric series in my papers

Several hypergeometric series appearing in my papers will be introduced and discussed.

Atul Dixit (Indian Institute of Technology Gandhinagar)

On values of the Riemann zeta function at odd positive integers

While the Riemann zeta function $\zeta(s)$ at even positive integers is known to be transcendental, the arithmetic nature of the odd zeta values remains mysterious with only $\zeta(3)$ known to be irrational, thanks to Apéry. In his famous notebooks, Srinivasa Ramanujan obtained a beautiful formula for $\zeta(2m+1)$ which encodes fundamental transformation properties of Eisenstein series on $\mathrm{SL}_2(\mathbb{Z})$ and their Eichler integrals.

In a joint work with Bibekananda Maji, we extended a transformation by Kanemitsu, Tanigawa and Yoshimoto from 2001 for the generalized Lambert series $\sum_{n=1}^{\infty} \frac{n^{N-2h}}{\exp(n^N x)-1}$. Surprisingly, the same series is located on page 332 of Ramanujan's Lost Notebook in a slightly more general form. Our extension gives a beautiful new generalization of Ramanujan's formula for $\zeta(2m+1)$ and has applications in studying transcendence of certain important constants. In a subsequent joint work with Rahul Kumar, Rajat Gupta and Bibekananda Maji, we comprehensively studied the more general Lambert series $\sum_{n=1}^{\infty} \frac{n^{N-2h}\exp(-an^N x)}{1-\exp(-n^N x)}$, thereby extending our previous results. The latter study gives, for example, a new relation between $\zeta(3), \zeta(5), \zeta(7), \zeta(9)$ and $\zeta(11)$.

In a very recent joint work with Rajat Gupta, we have obtained a Ramanujan-type formula for $\zeta^2(2m+1)$. The crucial part in deriving such a result is to obtain the right analogues of Eisenstein series. This is achieved by extending some work of N. S. Koshliakov. This talk will be based on the results from these three joint works.

Amanda Folsom (Amherst College)

Quantum modular forms and applications

Quantum modular forms were defined in 2010 by Zagier; they are somewhat analogous to ordinary modular forms, but they are defined on the rational numbers as opposed to the upper half complex plane, and have modified transformation properties. In the last few years, quantum modular forms have been shown to be intimately connected to Ramanujan's mock theta functions, which emerged 90 years earlier in 1920. As of a few years ago, "...no one... ha[d] actually proved... that any of Ramanujan's mock theta functions are really mock theta functions according to his definition," observed Berndt in his article "Ramanujan, his lost notebook, its importance." In this talk, we will discuss these intertwined topics, the more general relationship between mock modular and quantum modular forms, and applications to other areas including combinatorics and topology. We will mention recent work by the speaker and others such as Bringmann-Rolen, Choi-Lim-Rhoades, Griffin-Ono-Rolen, and Hikami-Lovejoy.

Frank Garvan (University of Florida)

Mock theta functions: yesterday and today

We discuss historical and modern developments of Ramanujan's mock theta functions. This includes a cast of characters. We present a number of recent discoveries and identities found through computer experiments.

Michael Hirschhorn (University of New South Wales)

Some identities of Ramanujan, Andrews and Berndt

I will prove some identities of Ramanujan, Andrews and Berndt involving Ramanujan's continued fraction which appear in the Lost Notebook and Part I.

Soon-Yi Kang (Kangwon National University)

Generators of the space of harmonic weak Maass forms

Every modular function on the full modular group that is holomorphic away from the cusp at infinity can be written as a linear sum of Niebur Poincaré series. We investigate various properties of these series.

Byungchan Kim (SeoulTech)

Partial or false theta series in q-series

In his (lost) notebooks, Ramanujan recorded an asymptotic expansion for a partial theta series and many q-series identities involving partial or false theta series. Under Prof. Berndt's supervision, these results have given me great motivation to study partial or false theta series. In this talk, I will discuss their roles in q-series and related areas with emphasis on asymptotic behaviors and q-combinatorics.

Christian Krattenthaler (Universität Wien)

A joint central limit theorem for the sum-of-digits function, and asymptotic divisibility of Catalan-like sequences

It is well known that almost all central binomial coefficients $\binom{2n}{n}$ are divisible by any given power of 2, and it is also well known (but maybe less so) that the same is true for any given natural number. This result is a consequence of the asymptotic p-adic behaviour of the sum of digits function. We generalise this result to a multivariate central limit theorem. As application, we obtain that any integer sequence that is defined by a product/quotient of factorials (in a precise sense) has the same property as central binomial coefficients: given any positive integer m, almost all elements of such a sequence are divisible by m. This includes numerous sequences, such as multinomial coefficients, Catalan numbers and their generalisations and variations, and also various map counting sequences.

This is joint work with Michael Drmota.

Amita Malik (Rutgers University)

Two stopovers in BCB's mathematical journey: Riemann zeta and partitions

In the first part of the talk, we discuss certain results about the distributions of zeros of the derivatives of the Riemann ξ function. In the later part, we talk about combinatorics of some restricted partitions.

Victor Moll (Tulane University)

Valuations of integer sequences

For an integer x and a prime p, define $\nu_p(x)$ to be the p-adic valuation of x. A selection of examples of the sequence of valuations $\{\nu_p(x_n)\}$ is presented. These include Stirling numbers, the ASM numbers (counting alternating sign matrices), the involution numbers as well as the sequence $e_n = \int_0^\infty e^{-x} F_n(x)\, dx$ connected to the Fibonacci polynomials $F_n(x)$.

Ken Ono (Emory University)

The Jensen-Polya program for the Riemann Hypothesis and related problems

In 1927 Polya proved that the Riemann Hypothesis is equivalent to the hyperbolicity of Jensen polynomials for Riemann's ξ-function. This hyperbolicity has only been proved for degrees $d = 1, 2, 3$. For each d we prove the hyperbolicity of all but (perhaps) finitely many Jensen polynomials. Moreover, we establish outright hyperbolicity for all $d < 10^{26}$. We obtain a general theorem which models such polynomials by Hermite polynomials. This theorem also allows us to prove a conjecture of Chen, Jia, and Wang on the partition function. This result can be thought of as a proof of GUE for the Riemann zeta function in derivative aspect. This is joint work with Michael Griffin, Larry Rolen, Jesse Thorner and Don Zagier.

Peter Paule (Research Institute for Symbolic Computation, Linz)

Ramanujan, Weierstrass, and Computer Algebra

The talk reports on recent progress concerning computer algebra tools related to q-series and modular functions. A major part of the talk is devoted to a proof of the Weierstrass gap theorem which, in contrast to standard approaches, does not make use of the Riemann-Roch formula. The results presented arose in joint work with Silviu Radu (RISC).

Michael Schlosser (University of Vienna)

The bilateral afterlife of Srinivasa Ramanujan

Many nice and deep results are connected to Ramanujan. In the theory of basic hypergeometric series, this for instance applies to the ${}_1\psi_1$ summation, a bilateral series identity which Ramanujan wrote down without providing a proof. Only twenty years after Ramanujan's premature death this result was published (by Hardy) and only after nine more years the first proof was found (by W. Hahn). Even more intriguing examples are the two Rogers–Ramanujan identities, which again were found by Ramanujan but in this case had already much earlier been found and proved by Rogers. They are particularly inspiring and connect to various areas of mathematics. While some people refer to the two Rogers–Ramanujan identities as RR1 and RR2, in this talk I will use the acronym RNDT (Ramanujan's nice deep theorems). I will highlight some recently discovered *bilateral extensions* of RNDT, which in this talk I shall refer to as BERNDT. I will discuss different proofs of BERNDT and present similar results (relatives of BERNDT) including infinite families of BERNDT.

Andrew Sills (Georgia Southern University)

Rogers–Ramanujan type identities, old and new

We shall take a tour of Rogers–Ramanujan type identities. The pre-history traces back to Euler in the 1740's and new identities of this type continue to be discovered in 2019.

Dennis Stanton (University of Minnesota)

Comments on cranks and the rank

Ramanujan's congruences mod 5, 7, and 11 for the partition function $p(n)$ may be proven combinatorially using cranks and the rank. Explicit bijections for the core-crank classes exist, but not for the Andrews-Garvan crank or the Dyson rank. Some stronger combinatorial conjectures are given for each of these cranks and the rank.

Armin Straub (University of South Alabama)

On the Gaussian binomial coefficients, the simplest of q-series

In the early 90s, Loeb showed that a natural extension of the usual binomial coefficient to negative (integer) entries continues to satisfy many of the fundamental properties. In particular, he gave a uniform binomial theorem as well as a combinatorial interpretation in terms of choosing subsets of sets with a negative number of elements. We tell this remarkably little known story and show that all of it can be extended to the case of Gaussian binomial coefficients.

This talk is based on joint work with Sam Formichella.

Liuquan Wang (Wuhan University and NCTS)

Representations of mock theta functions

Motivated by the works of Liu, we provide a unified approach to find Appell-Lerch series and Hecke-type series representations for mock theta functions. We establish a number of parameterized identities with two parameters a and b. Specializing the choices of (a, b), we not only give various known and new representations for the mock theta functions of orders 2, 3, 5, 6 and 8, but also present many other interesting identities. We find that some mock theta functions of different orders are related to each other, in the sense that their representations can be deduced from the same (a, b)-parameterized identity.

This is based on a joint work with Dandan Chen.

Ole Warnaar (University of Queensland)

Some (q-)integrals Ramanujan would have liked

Ramanujan was a master in evaluating hypergeometric integrals and q-integrals. In this talk I will discuss a number of such integral evaluations that naturally arise in string theory and conformal field theory.

Kenneth Williams (Carleton University)

Ramanujan and ternary quadratic forms

Ramanujan's contribution to ternary quadratic forms and a survey of where it has led.

Ae Ja Yee (Penn State University)

Singular overpartitions and truncated partition theorems

In the 1970s, Andrews obtained various interesting Rogers-Ramanujan type partition identities using partition sieves. His sieve methods and results were generalized by many mathematicians including Bressoud, Gessel and Krattenthaler. Recently, such partition identities have received a lot of attention in the work of Andrews on singular overpartitions and the truncated pentagonal number theorem. In this talk, I will first discuss the combinatorics of singular overpartitions and some new discoveries. I will also discuss further truncated partition theorems that are reminiscent of Andrew's truncated pentagonal number theorem.

This talk is based on joint work with S. Seo and C. Wang.

Abstracts of Contributed Talks

Zafer Selcuk Aygin (University of Lethbridge)

Ramanujan's convolution sum twisted by Dirichlet characters

We find formulas for convolutions of sum of divisor functions twisted by the Dirichlet character $\left(\frac{-4}{*}\right)$, which are analogous to Ramanujan's formula for convolutions of the usual sum of divisor functions. We use the theory of modular forms to prove our results. This work was completed when the author was a post-doctoral fellow at Nanyang Technological University, and is joint work with Nankun Hong.

Nilufar Mana Begum (Tezpur University)

Generating functions and congruences for partition functions related to third order mock theta functions

Recently, Andrews, Dixit and Yee in their paper "Partitions associated with the Ramanujan/Watson mock theta functions $\omega(q), \nu(q)$ and $\phi(q)$" published in *Res. Number Thoery*, 2015, introduced partition functions associated with Ramanujan/Watson third order mock theta functions $\omega(q)$ and $\nu(q)$. We find several new exact generating functions for those partition functions as well as the associated smallest parts functions and deduce several new congruences modulo powers of 5. In particular, let $\overline{\text{spt}}_\omega(n)$ denote the number of smallest parts in the overpartitions of n in which the smallest part is always overlined and all odd parts are less than twice the smallest part. We find the generating functions of $\overline{\text{spt}}_\omega(10n+5)$ and $\overline{\text{spt}}_\omega(50n+25)$. We also find several congruences modulo powers of 5 and conjecture some infinite families of congruences modulo powers of 5.

Paul Beirne (University College Dublin)

Knots, the colored Jones polynomial and stability

In 2006, Dasbach and Lin observed stability in the coefficients of the Nth colored Jones polynomial for alternating knots. This observation and its consequences have sparked a flurry of activity in both number theory and quantum topology. For example, Garoufalidis, Le and Zagier conjectured identities which have a striking resemblance to those occurring in the classical setting of Rogers and Ramanujan. In this talk, we discuss these developments and higher order stability for an infinite family of pretzel links.

This is partly joint work with Robert Osburn.

Gaurav Bhatnagar (University of Vienna)

On Entry II.16.12: A continued fraction of Ramanujan

G. H. Hardy remarked: "There is always more in one of Ramanujan's formulae than meets the eye, as anyone who gets to work to verify those which look the easiest will soon discover." Indeed, the verification of the beautiful continued fraction evaluation which is Entry 12 has proved to be most troublesome. The first proof (in 1985) was given by Adiga, Berndt, Bhargava and Watson, who further acknowledged the help they received from R. A. Askey and D. M. Bressoud. The proof is rather complicated. A few years later, Jacobsen (1989) clarified and simplified this proof. Ramanathan (1987) gave a formal proof, via contiguous relations.

In a previous study, the author (2014) showed that an elementary method due to Euler can be used to derive all of Ramanujan's q-continued fractions, except for one notable exception. The exception was Entry 12, of course.

In this talk, we provide two proofs. The first proof is by Euler's method and requires the usual manipulatorics expected from Ramanujan's results, but there are challenges to prove convergence. The second proof (from the theory of orthogonal polynomials) gives convergence for free, but there are some challenges here too, which require the first proof to resolve.

All in all, we can certainly say that there is more than meets the eye in this formula of Ramanujan. This is joint work with Mourad Ismail.

Arunabha Biswas (University of North Texas)

Search for higher Mahler measure of cyclotomic polynomials.

We study higher Mahler measure which is a generalization of classical Mahler measure. Apart from giving a quick overview of Mahler measure and higher Mahler measure, in this talk we shall propose a possible scheme to calculate higher Mahler measures of cyclotomic polynomials involving the beta function and π.

Aubrey Blecher (University of Witwatersrand)

Distinct tuples in integer partitions

We study the total number of distinct (sequential) r-tuples in partitions of n. These statistics generalise the number of distinct parts in a partition. In the early part of this paper we develop the generating functions for small values of r. Then we use these methods in the case of general r-tuples. We find the average number of distinct r-tuples for fixed r, as $n \to \infty$. Finally we show that as n grows, the total number of distinct r-tuples (normalised by q^r) converges to an explicitly determined "enveloping" power series.

Douglas Bowman (Northern Illinois University)

Euler combinatorics for divergent continued fractions after Ramanujan

We give Euler-type combinatorial descriptions of the terms and coefficients of classical numerators and denominators of a general class of divergent continued fractions which converge in residue classes modulo three. The descriptions involve finite integer sequences with difference and congruence restrictions. Special cases are considered and connections with a number of counting sequences are described. For example, new combinatorial formulas for Fibonacci and Pell numbers arise that demonstrate certain known congruence properties for these sequences.

Walter Bridges (Louisiana State University)

Kadell-type partition inequalities and applications to sum-product conjectures of Kanade-Russell

The Rogers-Ramanujan identities state that

$$\sum_{n\geq 0} \frac{q^{n^2}}{(q;q)_n} = \frac{1}{(q,q^4;q^5)_\infty} \qquad \text{and} \qquad \sum_{n\geq 0} \frac{q^{n^2+n}}{(q;q)_n} = \frac{1}{(q^2,q^3;q^5)_\infty}.$$

The difference in the left and right sums clearly has nonnegative coefficients, but without reference to the Rogers-Ramanujan identities it is not obvious that the same is true for the difference in product sides. In 1998, Kadell provided a combinatorial proof, and since then many other differences of products have been considered. We give combinatorial proofs and extensions of partition inequalities due to Berkovich-Garvan and McLaughlin. These may be applied to recently conjectured sum-product identities of Kanade-Russell. We show nonnegativity of differences of corresponding product sides, which provides some evidence as to the truth of these conjectures.

Hannah Burson (University of Illinois)

Simultaneous core partitions into k distinct parts

In 2016, using a combinatorial bijection with certain abaci diagrams, Nath and Sellers enumerated $(s, ms\pm 1)$-core partitions into distinct parts. In this talk, we explain new generalizations of this theorem, with a focus on a generating polynomial that enumerates simultaneous core partitions by the number of parts.

This work is joint with Simone Sisneros-Thiry and Armin Straub.

Zhu Cao (Kennesaw State University)

Exact covering systems, quadratic forms, and product identities for theta functions

In this talk, we discuss the connections among exact covering systems, quadratic forms, and product identities for theta functions. We show that most of the identities among Ramanujan's forty identities for the Rogers-Ramanujan functions can be proved using ECS and quadratic forms. We also work on ternary quadratic forms and present a list of new identities for the product of more than two theta functions.

Tsz Chan (University of Memphis)

On the congruence equation $\overline{a}+\overline{b}\equiv\overline{c} \pmod p$

In this talk, we will consider the congruence equation $\overline{a}+\overline{b}\equiv\overline{c} \pmod p$ with $1\leq a,b,c\leq H$ where $\overline{x}$ stands for the multiplicative inverse of $x \pmod p$. We prove that its number of solutions is asymptotic to H^3/p when $H > p^{2/3+o(1)}$ by estimating a certain average of Kloosterman sums via Gauss sums. On the other hand, when $H < p^{1/2}\sqrt{\log p}$, the number of solutions has order of magnitude at least $H\log H$. It would be interesting to understand better its transition of behavior. By transforming the question slightly, one can relate the problem to a certain first moment of Dirichlet L-functions at $s=1$.

Sunil Chebolu (Illinois State University)

Mersenne primes and Fermat primes

Fuchs' problem is the problem of characterizing abelian groups which occur as the group of units of a commutative ring. I will present a solution to this problem for the class of indecomposable abelian groups. As a byproduct we will get some new characterizations of Mersenne primes and Fermat primes.

Shane Chern (Penn State University)

Weighted partition rank and crank moments and Andrews-Beck type congruences

Recently, George Beck conjectured and George Andrews proved a number of interesting congruences concerning Dyson's rank function and Andrews–Garvan's crank function. In this talk, we shall discuss two identities involving the weighted rank and crank moments, from which we may deduce more congruences of Andrews–Beck type.

Hyunsoo Cho (Yonsei University)

A bijection between self-conjugate and ordinary partitions and counting simultaneous cores as its application

We give a bijection between the set of self-conjugate partitions and that of ordinary partitions. Also, we show the relation between hook lengths of self conjugate partitions and corresponding partitions via the bijection. As a corollary, we give new combinatorial interpretations for the Catalan number and the Motzkin number in terms of self-conjugate simultaneous core partitions.

Madeline Dawsey (Emory University)

Integer partitions and a conjecture of John Thompson

Inspired by Hilbert's 12th problem, G. R. Robinson and J. G. Thompson studied the number fields obtained from character values of alternating groups. For a finite group G, let $K(G)$ denote the field generated over $\mathbb{Q}$ by its character values. For $n > 24$, they proved that

$$K(A_n) = \mathbb{Q}\left(\{\sqrt{p^*} \ : \ p \leq n \text{ an odd prime with } p \neq n-2\}\right),$$

where $p^* := (-1)^{\frac{p-1}{2}}p$. Confirming a conjecture of Thompson, we show that arbitrary suitable multiquadratic fields are similarly generated by the values of A_n-characters restricted to elements whose orders are only divisible by ramified primes. Our proof makes use of partitions of integers into distinct parts which are squares of π-numbers.

This is joint work with Ken Ono and Ian Wagner.

Karl Dilcher (Dalhousie University)

Infinite products involving Dirichlet characters and cyclotomic polynomials

Using some basic properties of the gamma function, we evaluate a simple class of infinite products involving Dirichlet characters as a finite product of gamma functions and, in the case of odd characters, as a finite product of sines. As a consequence we obtain evaluations of certain multiple L-series. We also derive expressions for infinite products of cyclotomic polynomials, again as finite products of gamma or of sine functions.

This is joint work with Christophe Vignat.

Alexander Dunn (University of Illinois)

The twisted second moment of modular half integral weight L–functions

Given a half-integral weight holomorphic Kohnen newform f on $\Gamma_0(4)$, we prove an asymptotic formula for large primes p with power saving error term for $\sum^*_{\chi \pmod p} \left|L(1/2, f, \chi)\right|^2$. Our result is unconditional, it does not rely on the Ramanujan–Petersson conjecture for the form f. There are two main inputs. The first is a careful spectral analysis of a highly unbalanced shifted convolution problem involving the Fourier coefficients of half-integral weight forms. This analysis draws on some of the ideas of Blomer–Milićević in the integral weight case. The second input is a bound for a short sum involving a product of Salié sums, where the summation length can be below the square root threshold. Half–integrality is fully exploited to establish such an estimate. We use the closed form evaluation of the Salié sum to relate our problem to the sequence $\alpha n^2 \pmod 1$. Our treatment of this sequence is inspired by work of Rudnick–Sarnak and Zaharescu on the local spacings of αn^2 modulo one.

Dennis Eichhorn (UC Irvine)

The differences between $p(n)$, $p(2n)$, and $p(2n+1)$

In many ways, the sequences $p(2n)$ and $p(2n+1)$ are each more well poised than $p(n)$ itself. Several empirical observations highlight this fact, and in particular the iterated differences of these sequences are somewhat surprising. The behavior of various statistics on the sets of partitions these sequences count are also of interest. A priori, it is not obvious that these partition statistics should behave differently according to the parity of the number being partitioned; however, several of them do.

Both analytic heuristics and a new combinatorial proof will be included.

Larry Ericksen (Cornell)

Properties and polynomials related to restricted binary overpartitions

Binary overpartitions are partitions whose parts are powers of two and the first occurrence of any part may be overlined. Polynomials which encode these overpartitions will be identified by generating functions, recursions, and explicit formulas. We will also present various identities and congruences for these polynomial sequences. Further extensions will be discussed.

This is joint work with Karl Dilcher.

Susan Ficken (Anne Arundel Community College)

Asymptotics of the zeros of Fine's function

We give asymptotics of the zeros of Nathan Fine's basic hypergeometric function $F(a, b; t)$ as $t \to \infty$. We describe the main result and outline the proof, which employs Rouché's theorem along with suitable comparison functions.

Craig Franze (The Ohio State University)

An asymptotic expansion related to the Dickman function

If one wishes to estimate the number of integers free of large prime factors, then ultimately one is lead to a study of the Dickman function, $\rho(u)$. This function can be written as an alternating sum of integrals,

$$\rho(u) = \sum_{\ell=0}^{\infty} (-1)^{\ell} K_{\ell}(u),$$

where

$$K_{\ell}(u) = \underset{\substack{1 \le t_1 < \cdots < t_{\ell} \le u \\ t_1 + \cdots + t_l \le u}}{\int \cdots \int} \frac{dt_1}{t_1} \cdots \frac{dt_{\ell}}{t_{\ell}}.$$

Soundararajan (2012) gave an asymptotic expansion for $K_{\ell}(u)$. In this talk, I will discuss a generalization of this expansion, as well as its computational implications.

Oscar Gonzales (University of Illinois)

An observation of Rankin on Hankel determinants

While studying the location of the zeros of Eisenstein series, Rankin considered the determinants Δ_n of an associated Hankel matrix. He observed that the first few possess remarkable factorizations: each of them is a highly composite number expressible as a product of powers of small primes. Here we give an explicit formula for Δ_n using work of Kaneko and Zagier on Atkin polynomials.

Chadwick Gugg (Georgia Southwestern State University)

Ramanujan's 40 identities for the Rogers-Ramanujan functions

In his notebooks, Ramanujan recorded without proof 40 modular identities for the Rogers-Ramanujan functions. In this talk, we discuss these identities and related identities involving powers of the Rogers-Ramanujan functions, analogues of the Rogers-Ramanujan functions, and the Rogers-Ramanujan continued fraction.

Lakshika Gunawardana (Southern Illinois University Carbondale)

A primitive 15–theorem?

In this study we are trying to determine whether there exists a finite set S of integers such that every positive definite integral quadratic form that primitively represents the integers in S, primitively represents all positive integers. We use the famous 204 list presented by Bhargava on representations and try to identify primitive universal forms in that list to come up with a conjecture.

Rajat Gupta (Indian Institute of Technology Gandhinagar)

A Ramanujan-type formula for $\zeta^2(2m+1)$ and its generalizations

A Ramanujan-type formula involving the squares of odd zeta values is obtained. The crucial part in obtaining such a result is to conceive the correct analogue of the Eisenstein series involved in Ramanujan's formula for $\zeta(2m+1)$. The formula for $\zeta^2(2m+1)$ is then generalized in two different directions, one, by considering the generalized divisor function $\sigma_z(n)$, and the other, by studying a more general analogue of the aforementioned Eisenstein series, consisting of one more parameter N. A number of important special cases are derived from the first generalization. For example, we obtain a series representation for $\zeta(1+\omega)\zeta(-1-\omega)$, where ω is a non-trivial zero of $\zeta(z)$. We also evaluate a series involving the modified Bessel function of the second kind in the form of a rational linear combination of $\zeta(4k-1)$ and $\zeta(4k+1)$ for $k \in \mathbb{N}$.

This is joint work with Atul Dixit.

Heekyoung Hahn (Duke University)

Poles of triple product L-functions of monomial representations

Little is known about the order of poles of triple product L-functions in higher rank. In this talk we will investigate the case of monomial representations. One hopes that providing examples in this simple setting will be useful for developing intuition for the general case.

Bernhard Heim (German University of Technology in Oman)

Ramanujan's τ function and non-sign changes

Sign changes of Fourier coefficients of modular forms is an important topic in number theory, and became in the last 15 years a very active area of research. The talk reports on joint work and results with Markus Neuhauser and Robert Troeger. Applications towards $\tau(n)$ are given.

Wade Hindes (Texas State University)

Dynamical and arithmetic degrees of random iterates

We study the degree growth rate of (i.i.d) random sequences of dominant, rational self-maps on projective space. We then apply our results to height growth and height counting problems in random orbits.

Nankun Hong (Nanyang Technological University)

Rank function and Lambert series

In this talk, we will show how to represent the generating function of the rank function as a summation involving three parts, a constant, a Lambert series, and product parts. Furthermore, we can express two different Lambert series corresponding to generating functions of the rank modulo two integers as a summation of the same series with different coefficients. Some applications of this method will be provided.

Timothy Huber (University of Texas Rio Grande Valley)

A Ramanujan-Sato series primer

Much of the theory needed to derive Ramanujan-Sato series at each level is well known and beautifully presented in a number of works. However, no single reference exists that incorporates comprehensive theoretical and algorithmic detail allowing one to formulate complete classes of series at each level. This talk outlines such a guide. For each genus zero subgroup and a corresponding Hauptmodul, a uniquely determined modular form of weight two will be constructed that satisfies a third order differential equation whose polynomial coefficients are explicitly determined. An algorithm is outlined for the computation of a complete list of singular values of the Hauptmodul with fixed degree over Q in a fundamental domain. Singular values are established from modular equations satisfied by the Hauptmodul. Formulas for coefficients of Ramanujan-Sato series are given in terms of the singular values. This construction allows one to formulate all Ramanujan-Sato series in which the coefficients have fixed degree over Q.

Brad Isaacson (New York City College of Technology)

On some elementary character sum identities

We prove two formulas expressing character sums as a linear combination of generalized Bernoulli numbers. These sums are generalizations of sums first investigated by Berndt in connection with character sum identities of Apostol.

Uha Isnaini (National Institute of Education Singapore)

On k-restricted overpartitions

In this talk, we introduce k-restricted overpartitions, which are generalizations of overpartitions. In such partitions, among those parts of the same magnitude, one of the first k occurrences may be overlined. We first give the generating function and establish the 5-dissections of k-restricted overpartitions. Then we provide a combinatorial interpretation for certain Ramanujan type congruences modulo 5.

Paul Jenkins (Brigham Young University)

The arithmetic of modular grids

A modular grid is a pair of sequences f_m, g_n of weakly holomorphic modular forms where for all m and n, the nth coefficient of f_m is the negative of the mth coefficient of g_n. These grids were first noted by Zagier in weights 1/2 and 3/2 in the Kohnen plus space, and such grids have appeared for Poincaré series, for modular forms of integral weight, and in many other situations. We give a general proof of Zagier duality for canonical bases of spaces of weakly holomorphic modular forms of integral or half-integral weight and arbitrary level.

This is joint work with Michael Griffin and Grant Molnar.

Lin Jiu (Dalhousie University)

On harmonic sums: integral and matrix representations with connections to partition-theoretic generalization of the Riemann zeta-function and random walks

Inspired by the partition-theoretic generalization of the Riemann zeta-function, we study the harmonic sums, since the special cases of them are identical. The connection between the generating function and the Beta function leads to integral representations, including special zeta values. Further study on harmonic sums and random walk models yields matrix representations, which provide alternative proofs for some combinatorial identities, through diagonalization.

This is joint work with Diane Y.H. Shi.

Yeongseong Jo (University of Iowa)

Rankin-Selberg L-functions via Good Sections

The celebrated conjecture of Ramanujan states that all local components of an irreducible cuspidal automorphic representation of $GL(n)$ for some number field are tempered. If we can describe the existence of all symmetric k-th power lifts from $GL(2)$ to $GL(k+1)$, the conjecture is true for $n = 2$. For $k = 2$, Bump and Ginzburg established the integral representation yielding the desired L-functions for $GL(n)$ and the twisted version was recently constructed by Takeda. Unfortunately the local functional equation involves intertwining operators opposed to Fourier transforms appearing in the well-known Rankin-Selberg integral for $GL(n) \times GL(n)$. In this talk, we consider the modified local Rankin-Selberg integral to incorporate intertwining operators at finite ramified places. In order to define analogous L-functions, we adopt the notion of "Good Sections" introduced by Piatetski-Shapiro and Rallis. If time permits, we explain how this framework is relevant to eventually compute local symmetric square L-functions for $GL(n)$.

Edna Jones (Rutgers University)

Representations by ternary quadratic forms

Which integers can you represent by ternary quadratic forms? For example, can the integer 2019 be represented as the sum of a square, three times a square, and five times a square? A few kinds of representations over the integers (such as global representation and local representation) will be discussed. To better understand these representations, we will count how many solutions there are to congruences involving ternary quadratic forms using quadratic Gauss sums and Hensel's lemma.

Through its use of Gauss sums, this project first introduced me to the work of Bruce Berndt.

Pin Hung Kao (Flagler College)

Almost-prime values of reducible polynomials at prime arguments

We adopt A. J. Irving's double-sieve method to study the almost-prime values produced by products of irreducible polynomials evaluated at prime arguments. We will present a refinement of the double-sieve method and the improvements on Irving's results. We will also show that by combining estimates involving the upper and lower Diamond-Halberstam-Richert sifting functions, one can improve upon the results cited in Diamond-Halberstam for both small and large values of g, where g is the degree of the polynomial in question.

This is a joint work with Craig Franze.

William Keith (Michigan Technological University)

(k, j)-colored partitions: congruences and other behaviors

Congruences for partition generating functions that happen to be modular forms can be established with analytic techniques specific to that field, but Bruce Berndt and collaborators have written on more elementary tools that Ramanujan used or might have used to prove related theorems. Partitions into k colors where at most j can be used per size of part exhibit congruences that can be proved through such elementary means, as well as other behaviors that can be so illustrated.

Louis Kolitsch (University of Tennessee at Martin)

A pair of congruences for $(3, 5)$-regular partitions modulo 5

In this talk, $(3, 5)$-regular partitions will be defined and it will be shown that the number of $(3, 5)$-regular partitions of $25n + 12$ and $25n + 22$ are congruent to 0 (mod 5).

Shin-ya Koyama (Toyo University)

Applications of Ramanujan's method on the behavior of Euler products to Selberg zeta functions

For any congruence subgroup of the modular group, we extend the region of convergence of the Euler product of the Selberg zeta function beyond the boundary $\Re s = 1$, if it is attached with a nontrivial irreducible unitary representation. The region is determined by the size of the lowest eigenvalue of the Laplacian, and it extends to $\Re s \geqslant 3/4$ under Selberg's eigenvalue conjecture. More generally, for any unitary representation we establish the relation between the behavior of the partial Euler product in the critical strip and the estimate of the error term in the prime geodesic theorem. For the trivial representation, the proof essentially exploits the idea of the celebrated work of Ramanujan.

Brandt Kronholm (University of Texas Rio Grande Valley)

A technique in partitions

In a 1974 paper titled *A Technique in Partitions*, H. Gupta describes "...a remarkably simple method of dealing with generating functions..." with the goal of establishing closed-term formulas for partition functions restricted to parts from a finite set. Gupta writes "...this leads to a formula which establishes a perfect relationship between the number of partitions and a linear combination of certain combinatory functions in which the coefficients are nonnegative integers." The formula that Gupta describes is known as a constituent of a quasipolynomial. The main result of Gupta's paper shows that the sums of these coefficients from any constituent from a given quasipolynomial are the same.

In this talk we first revisit Gupta's proof in order to bring needed generality to it. We then make use of the result and establish several results, some of which are familiar and others that are new.

Rahul Kumar (Indian Institute of Technology Gandhinagar)

Zeros of combinations of the Riemann Ξ-function and the confluent hypergeometric function on bounded vertical shifts

In 1914, Hardy proved that infinitely many non-trivial zeros of the Riemann zeta function lie on the critical line using the transformation formula of the Jacobi theta function. Recently Dixit obtained an integral representation involving the Riemann Ξ-function and the confluent hypergeometric function linked to the general theta transformation. Using this result, we show that a series consisting of bounded vertical shifts of a product of the Riemann Ξ-function and the real part of a confluent hypergeometric function has infinitely many zeros on the critical line, thereby generalizing a previous result due to Dixit, Roy, Robles, and Zaharescu. The latter itself is a generalization of Hardy's theorem.

This is joint work with Dixit, Maji, and Zaharescu.

Debanjana Kundu (University of Toronto)

On the relation between the classical Iwasawa $\mu = 0$ conjecture and the Coates-Sujatha Conjecture A

In this paper, we improve upon a result by Coates and Sujatha and prove an equivalence of the classical $\mu = 0$ conjecture with Conjecture A of Coates and Sujatha. By doing so, we can fully resolve Conjecture A for Abelian extensions of $\mathbb{Q}$. Assuming finiteness of the Shafarevich-Tate group, our main result gives evidence for the classical $\mu = 0$ conjecture for density 1 primes.

Kagan Kursungoz (Sabanci University)

Andrews-Gordon type series for Schur's partition identity

We construct an evidently positive multiple series as a generating function for partitions satisfying the multiplicity condition in Schur's partition theorem. Refinements of the series when parts in the said partitions are classified according to their parities or values mod 3 are also considered. Direct combinatorial interpretations of the series are provided.

Bibekananda Maji (Indian Institute of Technology Gandhinagar)

Recent developments in the theory of the restricted partition function $p(n, N)$ and $\mathrm{spt}(n, N)$

In this talk, we shall discuss a finite analogue of a recent generalization of an identity in Ramanujan's Notebooks. Differentiating it with respect to one of the parameters leads to a result whose limiting case gives a finite analogue of Andrews's famous identity for $\mathrm{spt}(n)$, the number of smallest parts in all partitions of n. The latter motivates us to extend the theory of the restricted partition function $p(n, N)$, namely, the number of partitions of n with largest parts less than or equal to N, by obtaining the finite analogues of rank and crank for vector partitions as well as of the rank and crank moments. We shall discuss these and other related results in this talk.

This is joint work with Atul Dixit, Pramod Eyyunni and Garima Sood.

Nicholas Mayers (Lehigh University)

Integer partition statistics arising from seaweed algebras

Using the index theory of seaweed algebras, we explore various new integer partition statistics. We find relations to some well-known varieties of integer partitions as well as a surprising periodicity result.

James McLaughlin (West Chester University)

Some observations on Lambert series, vanishing coefficients and dissections of infinite products and series

Andrews and Bressoud, Alladi and Gordon, and others, have proven, in a number of papers, that the coefficients in various arithmetic progressions in the series expansions of certain infinite q-products vanish. In this talk it is shown that these results follow automatically (simply by specializing parameters) in an identity derived from a special case of Ramanujan's ${}_1\psi_1$ identity.

Likewise, various authors have proven results about the m-dissections of certain infinite q-products using various methods. It is likewise shown that many of these m-dissections also follow automatically from this same identity alluded to above.

It is also shown how applying similar ideas to certain other Lambert series gives rise to some rather curious q-series identities, such as, for any positive integer m,

$$\frac{\left(q, q, a, \frac{q}{a}, \frac{bq}{d}, \frac{dq}{b}, \frac{aq}{bd}, \frac{bdq}{a}; q\right)_\infty}{\left(b, \frac{q}{b}, d, \frac{q}{d}, \frac{a}{b}, \frac{bq}{a}, \frac{a}{d}, \frac{dq}{a}; q\right)_\infty} = \sum_{r=0}^{m-1} q^r \frac{\left(q^m, q^m, aq^{2r}, \frac{q^{m-2r}}{a}, \frac{bq^m}{d}, \frac{dq^m}{b}, \frac{aq^m}{bd}, \frac{bdq^m}{a}; q^m\right)_\infty}{\left(bq^r, \frac{q^{m-r}}{b}, dq^r, \frac{q^{m-r}}{d}, \frac{aq^r}{b}, \frac{bq^{m-r}}{a}, \frac{aq^r}{d}, \frac{dq^{m-r}}{a}; q^m\right)_\infty}$$

and

$$(aq;q)_\infty \sum_{n=1}^{\infty} \frac{na^n q^n}{(q;q)_n} = \sum_{r=1}^{m} (aq^r;q^m)_\infty \sum_{n=1}^{\infty} \frac{na^n q^{nr}}{(q^m;q^m)_n}.$$

Shashika Petta Mestrige (Louisiana State University)

Congruences for the partition function $p_{[1^c \ell^d]}(n)$

In this talk we prove infinite families of congruences for the partition function $p_{[1^c \ell^d]}(n)$ modulo powers of ℓ ($\ell = 5, 7, 11$) for any integers c and d. This partition function is a special case of a generalized partition function $p_\pi(n)$, which is defined by

$$\sum_{n=o}^{\infty} p_\pi(n) q^n = \prod_{m=1}^{\infty} \frac{1}{(1-q^{t_1 m})^{r_1}(1-q^{t_2 m})^{r_2} \cdots (1-q^{t_k m})^{r_k}}.$$

Here $\pi := \left[t_1^{r_1} t_2^{r_2} \cdots t_k^{r_k}\right]$ and t_i are positive integers and r_i are integers. To prove these congruences, we use Hecke operators, an explicit basis of the vector space of modular functions of the congruence subgroup $\Gamma_0(\ell)$ and the work of Atkin and Gordon on proving congruences for the partition functions $p_{-k}(n)$.

Mojtaba Moniri (Normandale Community College)

Pairs of relatively long, mod 4 disjoint consecutive runs under Mahler's recurrence

For the sequence $f_0 = 1, f_{n+1} = \lceil \frac{3}{2} f_n \rceil$, we first present relatively long runs of same parity numbers. E.g., for indices below 1,200,000 the longest run of odd numbers is of length 17, there are two such, and they start at $f_{260,599}$, respectively $f_{947,610}$. For runs of even terms with index below 1,200,000, the longest is again of length 17 and is unique in that range. This run starts with $f_{994,356}$.

If we start with an initial value $f_0 = m$, the length of the initial odd run is the highest j with $2^j|(m+1)$. For $j \geq 1$, the last term of this run is $\equiv 1$ mod 4 and all others (when $j \geq 2$) are $\equiv 3$ mod 4. So the first term of a length-k run of odd numbers is $\equiv -1$ mod 2^k. Clearly, length-k even runs have their first term $\equiv 0$ mod 2^k.

Next we consider mod 4, and long runs of numbers $\not\equiv 3$. For $f_0 = m$, let $M(m)$ be the length of the longest such run (anywhere in the sequence), or ∞ if it has arbitrarily long such runs. Infinite runs avoiding such a congruency class (or another 'forbidden' class mod a higher power of 2) may not exist if Mahler's conjecture on non-existence of Z-numbers holds. We illustrate $M(1) \geq 63$, witnessed twice before index 1,200,000 at terms f_{357174} to f_{357236} and f_{789273} to f_{789335}.

We present an m such that $M(m) \geq 112$, witnessed without any exceptional terms at the beginning. It does not lead to any higher run through index 1,200,000. We note that Dubickas and Mossinghoff (2009) showed in their systematic and much more computationally expensive work that for $m \leq 2^{57}$, the maximum it takes to reach a number $\equiv 3$ mod 4 is 137 steps and they presented such an initial value (easy to check once given).

We provide initial values giving relatively long runs of numbers $\equiv 3$ mod 4 immediately followed by a relatively long run of numbers $\not\equiv 3$ mod 4. Our examples for this type of congruency separation include blocks of the lengths (156,51), (137,54), and (100,59). We also show examples where the two groups are separated by a relatively short block breaking the pattern(s), e.g. with lengths (142,[3],58).

Seyyed Hamed Mousavi (Georgia Institute of Technology)

Pentagonal number theorem and zeroes of the Riemann zeta function

We begin this talk by using Rademacher's formula for $p(n)$ to state a generalization of the partition function of n to a a function of a real number x. In order to justify that this generalization is helpful, we prove a Pentagonal Number Theorem for the truncated version of Rademacher's formula. In particular, denoting the ith pentagonal number by G_i, and assuming RH, we prove that

$$\sum_{G_l < x} p(x - G_l) = O(\sqrt{p(x)}).$$

We use this equation to study the behavior of Chebyshev's function $\Psi(x)$. We prove that

$$\sum_{G_l < x} (-1)^l \Psi(e^{\frac{\pi}{6}\sqrt{24(x-G_l)-1}}) \left(\frac{1}{24(x-G_l)-1} - \frac{6}{\pi(24(x-G_l)-1)^{\frac{3}{2}}} \right) = O\left(\frac{e^{\frac{\pi(\frac{1}{2}+\delta)}{6}\sqrt{24x-1}}}{\sqrt{24x-1}} \right).$$

This fact gives us information about the distribution of zeros of the Riemann Zeta function, as we will discuss in this talk.

Hayan Nam (UC Irvine)

How to determine a partition up to conjugation using hook multisets

If two partitions are conjugate, their multisets of hook lengths are the same. Then one may wonder whether the multiset of hook lengths of a partition determines a partition up to conjugation. The answer turns out to be no. However, we may add an extra condition under which a given multiset of hook lengths determines a partition uniquely up to conjugation. Herman-Chung, and later Morotti found such a condition. In this talk, we give an alternative proof of Morotti's theorem and generalize it.

Jiakun Pan (Texas A&M University)

Quantum Unique Equidistribution conjecture for Eisenstein series in the level aspect

We study Eisenstein series on growing levels with general central characters, and find an asymptotic formula for their mass distribution. An interesting feature is that the main term depends on the values of the logarithmic derivative of Dirichlet L-functions on the 1-line. The estimation for the error terms uses the subconvexity bound of twisted L-functions by Blomer and Harcos. As a variation of the QUE conjecture raised by Rudnick and Sarnak, our research extends previous work of Kowalski, Michel, and Vanderkam, Holowinsky and Soundararajan, Nelson, Pitale, and Saha, and Koyama, among many other authors.

Bogdan Petrenko (Eastern Illinois University)

Some properties of nontrivial coefficients of cyclotomic polynomials

In 1987 Jiro Suzuki showed, by improving the argument of Issai Schur, that any integer is a coefficient of some cyclotomic polynomial. In this talk I will explain a recent generalization of this result due to Marcin Mazur (Binghamton University) and I.

Meenakshi Rana (Thapar Institute of Engineering and Technology Punjab)

Combinatorial interpretations of mock theta functions by attached weights

The work provides the combinatorial interpretations of twelve mock theta functions in terms of $(n+t)$–color partitions with attached weights. We further discuss combinatorial interpretations of the generalized versions of all these mock theta functions and present these into six families.

Bruce Reznick (University of Illinois)

Equal sums of cubes of quadratic forms.

About 20 years ago, I learned in one of Bruce Berndt's number theory seminars that Ramanujan proved in 1913 that

$$(3x^2+5xy-5y^2)^3+(4x^2-4xy+6y^2)^3+(5x^2-5xy-3y^2)^3=(6x^2-4xy+4y^2)^3.$$

We place this equation in context and completely solve $f_1^3+f_2^3=f_3^3+f_4^3$ for binary quadratic forms f_j. Often there is a third equal sum of two cubes. For example,

$$\begin{aligned}&(4x^2-4xy+6y^2)^3+(5x^2-5xy-3y^2)^3\\&=(6x^2-4xy+4y^2)^3-(3x^2+5xy-5y^2)^3\\&=(6x^2-8xy+6y^2)^3-(3x^2-11xy+3y^2)^3,\end{aligned}$$

$$\begin{aligned}&(4x^2-4xy+6y^2)^3+(3x^2+5xy-5y^2)^3\\&=(6x^2-4xy+4y^2)^3-(5x^2-5xy-3y^2)^3\\&=\left(\tfrac{94}{21}x^2-\tfrac{8}{21}xy+\tfrac{94}{21}y^2\right)^3+\left(\tfrac{23}{21}x^2-\tfrac{199}{21}xy+\tfrac{23}{21}y^2\right)^3.\end{aligned}$$

Herman Schaumburg (Northern Illinois University)

Partition identities encoded in Ramanujan's three limit continued fraction

We show four new integer partition identities by equating different formulas for the classical numerators and denominators of Ramanujan's three limit continued fraction. The relationship between these identities and the three limit continued fraction is analogous to the relationship between the Rogers-Ramanujan identities and the Rogers-Ramanujan continued fraction.

Robert Schneider (University of Georgia)

Eulerian series and the algebra of partitions

Much like the positive integers $\mathbb{Z}^+$, the set $\mathcal{P}$ of integer partitions ripples with interesting patterns and relations. Now, the prime decompositions of integers are in bijective correspondence with the set of partitions into prime parts, if we associate 1 to the empty partition. One wonders: might some number-theoretic theorems arise as images in $\mathbb{Z}^+$ (i.e. in prime partitions) of greater algebraic and set-theoretic structures in $\mathcal{P}$?

We show that many well-known objects from elementary and analytic number theory are in fact special cases of phenomena in partition theory: a multiplicative arithmetic of partitions that specializes to classical cases; a class of "partition zeta functions" containing the Riemann zeta function as well as countless exotic non-classical cases; and other phenomena spanning both additive and multiplicative number theory. We also discuss very recent ideas reaching for an algebraic theory of partitions.

Satyanand Singh (New York City College of Technology)

Generating fourth terms of Nathanson's lambda sequences

We consider the set

$$A_n = \cup_{j=0}^{\infty}\{\varepsilon_j(n) \cdot n^j : \varepsilon_j(n) \in \{0, \pm 1, \pm 2, \dots, \pm\lfloor n/2 \rfloor\}\}.$$

Let $\mathcal{S}_{\mathcal{A}} = \bigcup_{a \in \mathcal{A}} A_a$, where $\mathcal{A} \subseteq \mathbb{N}$. We denote by $\lambda_{\mathcal{A}}(h)$ the smallest positive integer that can be represented as a sum of h, and no less than h, elements in $\mathcal{S}_{\mathcal{A}}$. Nathanson studied the properties of the $\lambda_{\mathcal{A}}(h)$ sequence and posed the problem of finding the values of $\lambda_{\mathcal{A}}(h)$ which are very important in geometric group theory and additive number theory. When $\mathcal{A} = \{2, j\}$, we represent $\lambda_{\mathcal{A}}(h)$ by $\lambda_{2,j}(h)$.

For fixed $h \in \{1, 2, 3\}$, the values of $\lambda_{2,j}(h)$ are known as j runs over the odd integers bigger than 1. In this presentation, we extend this result further by illustrating how to generate $\lambda_{2,j}(4)$ as j runs over the odd integers bigger than 1. In particular we will show that $\lambda_{2,3}(4) = 150$ and $\lambda_{2,5}(4) = 83$. Our techniques involve establishing the insolubility of certain exponential diophantine equations.

Nicolas Smoot (Research Institute for Symbolic Computation, Linz)

Some new identities from the Ramanujan-Kolberg algorithm

Ramanujan's identities involving the generating functions for $p(5n+4)$ and $p(7n+5)$ are considered to be among his finest results. These were shown by Kolberg to be special cases of a larger class of relationships expressing generating functions for $p(mn + j)$ in terms of eta quotients. The form of these identities is prevalent throughout the theory of partitions. They are useful in the verification of families of partition congruences, as well as in the study of certain conjectures in the theory of modular functions. In 2014 Silviu Radu developed an algorithm to compute the Ramanujan-Kolberg identities inherent in various arithmetic functions. This algorithm has now been given a complete implementation. We will show some interesting examples found using the algorithm. We include some new results, as well as some interesting improvements on established partition congruences.

Fikreab Solomon (CUNY - The Graduate Center)

Zeta functions of nilpotent groups

The study of subgroup growth zeta functions is a relatively young research area. In my thesis, I consider nilpotent groups and I attempt to generalize the notion of the cotype zeta function of an integer lattice to finitely generated nilpotent groups. The Dirichlet coefficients of the zeta functions are expressed in terms of Hall-Littlewood polynomials. The zeta functions help in determining the distribution of subgroups of finite index and provide more refined invariants in the analytic number theory of nilpotent groups.

Garima Sood (Indian Institute of Technology Gandhinagar)

A finite analogue of the Beck-Chern theorem and some results on self-conjugate S-partitions

Recently, we obtained a finite analogue of a generalization of a beautiful q-series identity of Ramanujan given by Dixit and Maji. In this talk, we discuss an identity involving a finite analogue of $N_{SC}(n)$, the number of self-conjugate S-partitions counted with a certain specific weight. Such partitions were first studied by Andrews, Garvan and Liang. In order to do this, first we need to find a finite analogue of a result of Beck, recently proved by Chern. We shall also discuss our generalization of an identity of Garvan which is accomplished by first obtaining a finite analogue of Rogers-Fine identity and that of an identity in Ramanujan's Lost Notebook.

This is joint work with Atul Dixit, Pramod Eyyunni and Bibekananda Maji.

Jonathan Sorenson (Butler University)

Recent work on the Erdős-Selfridge function $g(k)$

Let $P(n)$ denote the largest prime divisor of a positive integer n, and let $g(k)$ denote the smallest integer larger than $k+1$ such that $P(\binom{g(k)}{k}) > k$. We have a new algorithm to compute $g(k)$, and use it to extend the computational results of Lukes, Scheidler, and Williams (1997), and in particular we have

$$g(270) = 17012\ 60056\ 85638\ 85052\ 85598.$$

Under an unproven but reasonable *uniform distribution heuristic*, we show that $\log g(k)$ is proportional to $k/\log k$.

This is joint work with Brianna Sorenson (undergraduate student) and Jonathan Webster, both at Butler University.

Emily Stamm (Allstate)

An investigation of (nearly) weakly primes

A prime number p is called a weakly prime in base b if when any single digit of the base b expansion of b is switched to any other digit $0, \ldots, b-1$, the resulting number is composite. Tao proved that for any fixed base b, a positive proportion of primes are weakly in base b. In this paper, we investigate the exact proportion of such primes for a given base b, and how the values of b and $\phi(b)$ affect this proportion. We provide two possible sieve formulations for this proportion. We also prove several propositions regarding weakly primes across bases and examine weakly primes in small prime bases. We also include an algorithm for generating weakly primes with constant space complexity, and numerical data generated by this algorithm.

Kenneth Stolarsky (University of Illinois)

Rogers-Szego polynomials and products of algebraic numbers

The construction and distribution of irreducible non-cyclotomic polynomials with a root that is the product of two of its other roots has been pursued for its own sake (e.g. by Drmota, Dubickas, Ma, Skalba, and others). We indicate how such polynomials (call them "product including" polynomials) arise naturally in the context of root asymptotics of Rogers-Szego polynomials. These are famously orthogonal on the unit circle with a theta function weight. We indicate how these product including polynomials generate other polynomials with the same or related properties. Much here remains conjectural.

Karen Taylor (Bronx Community College, CUNY)

Dirichlet series and generalized Hecke groups

Duke introduced the groups $G(A)$ which naturally correspond to an ideal class, A, of a real quadratic field. These groups generalize the Hecke groups $G(\lambda)$, $\lambda > 2$. In this talk, I will describe two problems, which have been solved for the Hecke groups, which I am currently working on generalizing. One problem, solved by Rosen, is the description of the elements of $G(\lambda)$ by continued fractions. The other is Knopp and Sheingorn's construction of modular integrals on $G(\lambda)$ with prescribed log-polynomial periods.

Pee Choon Toh (Nanyang Technological University)

On a symmetric identity of Ramanujan

In his paper "On the expression of a number as the sum of two squares", G.H. Hardy recorded a beautiful identity of Ramanujan, as well as its generalization, and described briefly their proofs. We will present some generalizations of Ramanujan's identity.

This is joint work with Heng Huat Chan.

Thomas Tran (Duke University)

Secondary terms in asymptotics for the number of zeros of quadratic forms over integers

Recently Getz gave a secondary term in the asymptotic for the number of zeros of quadratic forms in an even number of variables. I present work in progress on a generalization to an odd number of variables.

Ian Wagner (Emory University)

Schwartz functions and modular forms

The linear programming bounds of Cohn and Elkies put the sphere packing problem in dimensions 8 and 24 within reach. Recently, Viazovska explicitly constructed special functions using modular forms which led to the resolution of the sphere packing problem in dimensions 8 and 24. We study possible generalizations of Viazovska's work which can be used to attack sphere packing problems in other dimensions and other related problems. We construct a number of infinite families of Schwartz functions using modular forms, which are eigenfunctions of the Fourier transform.

Tanay Wakhare (University of Maryland, College Park)

Special classes of q-bracket operators

We study the q-bracket operator of Bloch and Okounkov when applied to functions additive over all parts of a partition. We use these expansions to derive convolution identities for the functions f and link both classes of q-brackets through divisor sums. As a result, we generalize Euler's classic convolution identity for the partition function and obtain an analogous identity for the totient function. As corollaries, we generalize Stanley's theorem as well as provide several new combinatorial results.

Sarah Wesley (inte Q)

Generalizations of Fine's iterative identities

We exhibit a number of new generalizations of Fine's iterative identities. Some of the identities can be given combinatorial proofs, such as our extension of the Rogers-Fine identity. Some applications and future directions are indicated.

Wen-Bin Zhang (Univ. of West Indies and UIUC)

Exact Wiener-Ikehara theorems and Beurling's generalized numbers

A function $f(x)$ defined on $[a, \infty)$ is said to be linearly slowly decreasing with index $\alpha \geq 0$ if

$$\liminf_{x\to\infty,\, y/x\to 1+} (x \log^\alpha x)^{-1}(f(y) - f(x)) \geq 0.$$

Nondecreasing functions and slowly decreasing functions are linearly slowly decreasing with index $\alpha = 0$, but the converse is not true. Let $F(x)$ be a real-valued Lebesgue measurable function with support in $[0, \infty)$ such that $\int_0^\infty e^{-\sigma x}|F(x)|\, dx < \infty$. Let $\Delta^{*m}(t)$ be the m-fold convolution of $\Delta_\lambda(t) := (1 - |t|/(2\lambda))^+/2$, $t \in \mathbb{R}$, with itself. We have the following exact Wiener-Ikehara theorem.

We have $\lim_{x\to\infty}(e^x x^\alpha)^{-1}F(x) = L/\Gamma(\alpha+1)$ if $F(\log u)$ is a linearly slowly decreasing function of u with index α and there exists a constant $\lambda_0 \geq 0$ and a positive integer $m \geq 1 + [\alpha]/2$ such that, for every $\lambda > \lambda_0$,

$$\frac{1}{y^\alpha}\int_{-\infty}^{\infty} \Delta_\lambda^{*m}(t)e^{ity}(G(\sigma+it) - G(\sigma'+it))\, dt$$

approaches 0 as σ, $\sigma' \to 1+$ uniformly for $y \geq y_0(\lambda)$, where

$$G(s) = \int_0^\infty e^{-sx}F(x)\, dx - \frac{L}{(s-1)^{\alpha+1}}.$$

Conversely, if $\lim_{x\to\infty}(e^x x^\alpha)^{-1}F(x) = L/\Gamma(\alpha+1)$, then all the conditions hold for all $\lambda > 0$ and every integer $m \geq 1+[\alpha]/2$. The classical Wiener-Ikehara theorem is a special case of the above theorem with a nondecreasing function $F(x)$. We also have a Wiener-Ikehara upper bound theorem and a Wiener-Ikehara lower bound theorem. As a preliminary application to the theory of Beurling generalized numbers, we show that $M(x) = o(x)$ and the Chebyshev upper bound are not equivalent.
