Illinois Number Theory Conference
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University of Illinois at Urbana-Champaign

Abstracts of Talks

Peter Jaehyun Cho (University of Toronto), *The strong Artin conjecture and number fields with large class numbers.*

Assuming the Generalized Riemann Hypothesis and the Artin conjecture for the Artin L-function, Duke obtained an upper bound of the class number of a totally real field of degree n whose normal closure has S_n as its Galois group. Again assuming the GRH and the Artin conjecture, he constructed totally real number fields whose Galois closure is S_n with the class number having the same size as the upper bound up to a constant. We prove, unconditionally, that there exists a totally real quartic field with arbitrarily large discriminant whose normal closure has S_4 as its Galois group and its class number is of the same size as Duke's upper bound up to a constant. The main ingredient is that the 3-dimensional representation of S_4 is the symmetric square of the 2-dimensional representation of $\widetilde{S_4}$. We also prove that the strong Artin conjecture for S_n implies the analogous result for the fields of degree n.

Anna Haensch (Wesleyan University), *Almost universal ternary sums of squares and triangular numbers.*

Let x be an integer and define $T_x = (x(x+1))/2$. In this talk, we determine a complete characterization of all triples (a, b, c) for which the form $ax^2 + bT_y + cT_z$ is almost universal. To do so, we rely on the arithmetic theory of quadratic forms, in particular the theory of primitive spinor exceptions and techniques developed by Chan and Oh.

Jingjing Huang (Penn State University), *Binary Egyptian fractions.*

In this talk, we will survey various results about the diophantine equation $a/n = 1/x + 1/y$ with a fixed and n variable, and in particular its number of solutions $R(n; a)$. One can show that the behavior of $R(n; a)$ resembles that of the divisor function $d(n)$ in many aspects. More precisely, we will investigate the first moment estimate of $R(n; a)$ and hence higher moments. Then we show that $R(n; a)$ satisfies a Gaussian distribution, which is an analog of a classical theorem of Erdős and Turan. Lastly, we study the exceptional set $E_a(N)$, namely the number of n up to N such that $R(n; a) = 0$. We will improve a result by Hofmeister and Stoll, in which it is shown that the $E_a(N) \ll N/(\log N)^{1/\phi(a)}$. We actually get an asymptotic formula for $E_a(N)$ instead of just an upper bound.

Youness Lamzouri (University of Illinois), *Prime number races.*

Assuming the Generalized Riemann Hypothesis and the Grand Simplicity Hypothesis, M. Rubinstein and P. Sarnak established that the set of real numbers x such that $\pi(x; q, a_1) > \cdots > \pi(x; q, a_r)$ has a positive logarithmic density $\delta_{q;a_1,\ldots,a_r}$. In this talk, I will present how an asymptotic formula for the densities $\delta_{q;a_1,\ldots,a_r}$ when $r \geq 3$ is used to derive some surprising consequences on prime number races with three or more competitors. I will also describe recent joint work with K. Ford and S. Konyagin concerning a construction of certain hypothetical zeros of Dirichlet L-functions off the critical line, called *barriers*, that would imply $\delta_{q;a_1,a_2} = 0$ for some races $\{q; a_1, a_2\}$.

Taiyu Li (Shandong University and Towson University), *Sums of almost equal squares of primes.*

I will talk about my recent work joint with Angel Kumchev on sums of almost equal squares of primes. For example, we can prove that every sufficiently large integer N congruent to 5 modulo 24 can be written as $N = p_1^2 + p_2^2 + p_3^2 + p_4^2 + p_5^2$, with $p_j - \sqrt{N/5} \leq U = N^{\frac{1}{2} - \frac{11}{200} + \varepsilon}$, where p_j are primes. This improves substantially the previous results in this direction.

Kit Ho Mak (University of Illinois), *Maximal curves and the subcover problem.*

In this talk, I will introduce the concept of maximal curves and the subcover problem. Then I will talk about my recent joint work with Iwan Duursma showing that certain families of maximal curves are not Galois subcovers of the Hermitian curve.

Mojtaba Moniri (Western Illinois University), *Transcendental reals of low Sturmian complexity.*

The continued fraction of a real number generally speaking yields its Sturmian representation (which keeps track of where jumps occur in integer parts of multiples of the number), and the latter yields the base-10 representation. For real numbers of sufficiently low complexity, the continued fraction may still be relatively more involved computationally compared to their base-10 or even the Sturmian representation. We present transcendental reals whose Sturmian sequence we build, and in a sense appear to be less complex than their continued fraction.

Paul Pollack (University of Illinois), *Parity of the multiplicative partition function.*

An additive partition of n is a way of writing n as an (unordered) sum of natural numbers. Let $p(n)$ denote the number of additive partitions of n. A difficult 1967 conjecture of Parkin and Shanks predicts that $p(n)$ is even half the time and odd half the time (in the sense of asymptotic density). Let $f(n)$ be the number of multiplicative partitions of n, where a multiplicative partition of n is a way of writing n as an unordered product of natural numbers > 1. I will discuss a proof that $f(n)$ is odd about 57 percent of the time.

George Purdy (University of Cincinnati), *Smooth number estimates with applications to stereo vision.*

A number is k-smooth if none of its prime factors are greater than k. In binocular vision you have two retinas R and R', each containing n points. If you know which points of R correspond to which points of R', then it is trivial to reconstruct the original three dimensional points. The matching is not necessarily unique and there are even examples for which all $n!$ matchings correspond to sets of n points in space. We asked the following question: For which m in the range $[1, m]$ is there an example of $n+n$ retinal points with exactly m solutions. We show that such m's are n-smooth and conclude that there are no more than 2^n of them.

Johann Thiel (University of Illinois), *Number theory, probability, and the American flag.*

The Union Jack is the top left corner of the American flag that contains one star for each state. With the potential addition of Puerto Rico as the 51st state, the United States faces the trouble of adding a new star to the flag. Garibaldi created a program which gives “nice” arrangements for stars on the Union Jack for 1 to 100 stars, with three exceptions. Using some results of Erdös and Ford, we will discuss how rare these exceptions really are as we increase the number of stars. This is joint work with Dimitris Koukoulopoulos.

Robert Vaughan (Penn State University), *Waring's problem, Beatty sequences, and related questions.*

In joint work with William Banks and Ahmet Muhtar Guloglu a very general form of the local to global principle is established and applied to a variety of additive questions. It also shown that in Waring's problem, if the summands are restricted to be the k-th powers of a Beatty sequence, then results of the same quality as in the classical problem can be obtained.

Sam Wagstaff (Purdue University), *How often does $2kp+1$ divide p^p-1?.*

We answer the question in the title for fixed k and variable (prime) p. Some results are old, like this one: If $q = 4p+1$ is prime, then q divides $p^p - 1$. Here is a new result: If $q = 16p+1$ is prime, then q divides $p^{2p} - 1$. These questions arose during a study of the period of the Bell numbers modulo a prime. This is joint work with Peter Montgomery and Sangil Nahm.

Robert Marshawn Walker (University of Illinois), *Deducing a q-analog of Fleck's Congruence.*

Fleck proved the congruence bearing his name in 1913. Pick a prime number p: when certain conditions obtain, Fleck's congruence asserts that a certain sum of signed binomial coefficients (Fleck's sum) is divisible by p raised to some nonnegative power. Of course, polynomial versions of the binomial coefficients exist: the q-binomial coefficients. What would happen if we replace the binomial coefficients in Fleck's sum with their "q-counterparts?" What form could the moduli take as we depart from the realm of the integers? These questions are both natural to ask and related, and this talk will provide some perhaps surprising answers in the form of deep patterns we have observed–and in all but one case, proven–within the last year.