

Some important discrete distributions

1. Binomial:

- **Parameters:** n (positive integer), p ($0 \leq p \leq 1$)
- **P.m.f.:** $p(k) = \binom{n}{k} p^k (1-p)^{n-k}$ ($k = 0, 1, 2, \dots, n$)
- **Expectation and variance:** $\mu = np$, $\sigma^2 = np(1-p)$
- **Arises as:** Distribution of *number* of successes in success/failure trials (“Bernoulli trials”).

2. Poisson:

- **Parameter:** $\lambda > 0$
- **P.m.f.:** $p(k) = e^{-\lambda} \frac{\lambda^k}{k!}$ ($k = 0, 1, 2, \dots$)
- **Expectation and variance:** $\mu = \lambda$, $\sigma^2 = \lambda$
- **Exponential series formula:** $\sum_{n=0}^{\infty} \frac{\lambda^n}{n!} = e^\lambda$
- **Arises as:** Distribution of number of occurrences of *rare* events, e.g., accidents, disasters.
- **Poisson approximation to binomial distribution:** A binomial distribution in which n is large and p correspondingly small (so that np is of moderate size) is approximately Poisson with parameter $\lambda = np$; i.e., if X has binomial distribution with parameters n and p as above, then $P(X = k) \approx e^{-\lambda} \lambda^k / k!$, where $\lambda = np$.

3. Geometric:

- **Parameter:** p ($0 < p < 1$)
- **P.m.f.:** $p(k) = (1-p)^{k-1} p$ ($k = 1, 2, \dots$)
- **Expectation:** $\mu = 1/p$.
- **Geometric series formula:** $\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}$ ($|r| < 1$)
- **Arises as:** Distribution of *time (trial) of first success* in a success/failure trial sequence.

4. Negative binomial:

- **P.m.f.:** $p(k) = \binom{k-1}{r-1} (1-p)^{k-r} p^r$, $k = r, r+1, \dots$
- **Expectation:** $\mu = r/p$. (This is exactly r times the expectation of a geometric distribution. In other words, the expected time of the r -th success is r times the expected time of the 1-st success.)
- **Arises as:** Distribution of *time (trial) of r -th success* in a success/failure trial sequence. Generalizes the geometric distribution, which corresponds to the case $r = 1$.

5. Hypergeometric:

- **Parameters:** N, m, n
- **P.m.f.:** $p(k) = \frac{\binom{m}{k} \binom{N-m}{n-k}}{\binom{N}{n}}$ ($k = 0, 1, \dots, n$)
- **Expectation:** $\mu = (m/N)n$. (In other words, the expected number of red balls in the sample is exactly the “fair share”, namely the proportion of red balls in the entire box (i.e., m/N) times the number of balls in the sample (i.e., n).)
- **Arises as:** The distribution of the number of red balls, if n balls are drawn without replacement from a box containing a total of N balls, of which m are red.