Conditional probabilities: examples and case studies

1. **False positives/negatives:** A certain deadly disease occurs in 1 percent of the population. A blood test for this disease has a 2 percent false positive rate, and a 5 percent false negative rate (i.e., 2 percent of those not having the disease test positive, and 5 percent of those having the disease test negative). Suppose you want to put your mind at ease and take the blood test.

   (a) If you test positive, how certain can you be that you actually have the disease?

   (b) If you test negative, how certain can you be that you do not have the disease?

   Hints: Work with the events P= "test positive", N = "test negative", D = "has disease." Use the complement rule for conditional probabilities and Bayes' Rule.

2. **Polling results:** This is an example of a Real World Problem, with Real Data. taken from an article published in USA Today on 2/15/2000, on the Republican presidential primary election. The article is reproduced on the back of this page, and also available online at `http://www.usatoday.com/news/opinion/e1168.htm` (just to prove that I'm not making this up!). The key data given in the article are the following. (Note that these numbers refer not to the total voting population, but only to those planning to vote in the Republican primary; hence, for example, the small percentage of Democrats.)

   A. Bush leads McCain 49% – 42%

   B. Among Republicans, Bush leads McCain 59% – 34%

   C. Independents support McCain 54% – 36%

   D. Democrats who intend to vote in the Republican primary support McCain over Bush 61% – 27%

   E. 60% of the voters will be Republicans, 34% Independents, and 6% Democrats.

   (a) *Without using A*, derive the numbers in A (i.e., 49% for Bush and 42% for McCain) from the other pieces of information B–E.

   (b) How many (i.e., what percentage) of the McCain supporters are Republicans? How many are Independents? How many are Democrats?

   Hints: Introduce the events M="McCain supporter", B="Bush supporter", R="Rebublican", D="Democrat", I="Independent". Then translate the given data B–E, as well as the probabilities sought in questions (a) and (b), into probabilities (ordinary or conditional) involving these events. Finally, using appropriate probability rules, derive the probabilities sought from those that were given.

3. **Three biased coins:** Suppose you have 3 coins that produce heads with probabilities 1/2, 1/3, and 1/4, respectively. The coins look all alike, so you cannot tell them apart. You pick one of the 3 coins at random and toss it.

   (a) What is the probability that the coin comes up heads?

   (b) Suppose the coin comes up heads. What is the probability that this coin is the fair one, i.e., the one that produces heads with probability 1/2?

   Hints: Work with the events H= "coin comes up heads" and C_1 = "picked coin 1", C_2 = "picked coin 2", C_3 = "picked coin 3", where coin 1 is the coin with 1/2 heads probability, coin 2 is one with 1/3 heads probability, and coin 3 is one with 1/4 heads probability. Use the Average Rule for the first part, and Bayes' Rule for the second part.

4. **Card trick:** You have three cards, one colored red on both sides, one colored blue on both sides, and one colored red on one side and blue on the other. You pick one card at random and place it flat on the table. Suppose the top side is red. What is the probability that the other side is red as well?

   Hints: Use Bayes' Rule; cf. Ross, Example 3k.