Midterm Exam 2 Study Guide

Exam Basics

- **Date/time.** The exam will be held **Wednesday, July 11, during the regular class period.** The exam will begin at 11 am, and you will have 60 minutes to complete the exam, with possibly some additional time tacked on at the end. (Note that the two hour class period on Wednesdays allows some flexibility in this regard.)

- **Exam Review/Q&A Session.** Monday, July 9, 6 - 7 pm, 143 Altgeld.

- **Rules.** No calculators, closed notes/books. The problems will be selected so as to keep any algebraic or numerical computations to a minimum.

- **Exam format and content.** The exam will be on the material covered in class July 2 through July 9 and in HW 4 and 5. The corresponding sections in the text are 3.1–3.6 and 4.2. See below for a detailed syllabus. Note that Chapter 2 will not be on this exam, nor will the material covered the day before the exam (Tuesday, 7/10).

There will be five problems, each split into several individual steps or parts, worth 100 points altogether. The problems will be of one of the following types:

- **Computational problems.** Most of the problems will be purely computational questions (“Find the general solution to ...”, “Solve the initial value problem ...”, “Find the Wronskian ...”). These will be comparable to homework problems of this type, but they won’t involve lengthy computations.

- **Conceptual questions, statements of definitions/theorems, and short proofs.** A few questions may involve stating key concepts and theorems (such as linearity or the superposition principle) covered in class (see below for a list), or giving short proofs involving these concepts. The proofs will be at the level of proofs given in class or in the last two problems of HW 4.

- **Suggestions for preparing for the exam.**
  - **Memorize relevant formulas.** There is a small number of formulas that you should memorize. See below for a list.
  - **Review key concepts from your class notes and the text.** This will prepare you for conceptual questions. Use the list below as your study guide.
  - **Review/redos homework problems.** The computational problems on the exam will be similar to problems that have come up in the homework, so reviewing these problems is an excellent way to prepare for this part of the exam. Suggested practice problems are listed below.

Exam Checklist

Computational Problems

- **Solving constant coefficient homogeneous linear equations (3.1, 3.3, 3.4, 4.2).** Find (1) the characteristic equation for this equation; (2) a fundamental set of solutions (three cases: distinct real roots, repeated real roots, complex conjugate roots); (3) the general solution; (4) the solution to a given initial value problem. Conversely, given a general solution or a characteristic equation, find a differential equation that has this solution or characteristic equation. (Example: 3.1:17 from HW 4). Describe the behavior of the solution as \( t \to \infty \) (e.g., goes to infinity, goes to 0, oscillation with decaying amplitude, steady oscillation). (Note: Graphing of solutions will not be required in the exam.)

  **Practice problems (all from HW 4/5):** 3.1:1,10,17; 3.3: 7,17,18; 3.4:1,18,19,37; 4.2:11,14.

- **Solving non-homogeneous linear equations (3.5, 3.6).** Find a particular solution by the method of undetermined coefficients or the method of variation of parameters. Use this to construct the general solution to the nonhomogeneous equation and the solution to a given initial value problem.

  **Practice problems:** 3.5:1,3,19; 3.6:1,9.
• **Computing Wronskians (3.2).** Calculate the Wronskian $W(y_1, y_2)$ using (1) the determinant formula and (2) Abel's Theorem (Theorem 3.2.7).

  **Practice problems:** 3.2:5, 21, 28, 29, 31.

• **Basic calculations with complex numbers (3.3, 4.2).** E.g., convert complex numbers to polar form and vice versa, using Euler’s formula.

  **Practice problems:** 3.3:1, 4; 4.2:1, 2

**NOTE:** The method of reduction of order (p. 169–170) will not be on this exam.

### Formulas you should memorize

- Euler’s formula for $e^{ix}$.
- **Determinant Formula for the Wronskian.**
- **Abel’s Formula for the Wronskian.** (Theorem 3.2.7)
- **Recipe for the “educated guess” in the undetermined coefficient method.** Know the general form of the “guess” for $y_p$ for the cases of polynomials, exponential functions, and sine/cosine functions. Know how to modify this guess if there is duplication with solutions to the homogeneous equation.
- **Variation of parameters solution.** Formula (28) on p. 190 (Theorem 3.6.1) for the general solution to a nonhomogeneous equation using the variation of parameters method.

### Concepts/theorems you should be familiar with

- **Existence/uniqueness theorem for second order linear equations (Theorem 3.2.1).** You should know the statement, but not the proof. (See Problem 3.2:9 in HW 5)

- **Linear versus nonlinear DE’s.** You should be comfortable with operator notation for DE’s, know what it means for a differential operator to be linear, and be able to distinguish linear from nonlinear DE’s. (See class notes and Problem 3.2.14 in HW 5)

- **Superposition Principle for solutions to homogeneous linear differential equations.** You should know the statement of this principle and know when it applies. You should also be able to derive the principle using the linearity property. (See class notes and Theorem 3.2.2)

- **Superposition Principle for solutions to nonhomogeneous linear differential equations.** You should know the statement of this principle and when it applies, and be able to derive it. (See class notes.)

- **General solution to homogeneous second order linear differential equation.** You should know the form of the general solution to a homogeneous second order linear DE, but not the proof. (See Theorem 3.2.4)

- **General solution to nonhomogeneous second order linear equation.** Be familiar with the general structure of the solution, consisting of a particular solution of this nonhomogeneous equation, and the general solution to the corresponding homogeneous equation. You should be able to derive this form from the results about homogeneous linear equations and the superposition principle. (See class notes and Theorem 3.5.2)

### Questions?

If you have further questions (do we need to know X?), ask after class, at Monday’s Review Session, or just email me (ajh@illinois.edu).

**Good luck on the exam!**