Math 408, Spring 2008
HW Assignment 6, due Friday, 2/29/2008

Name (print please):

Instructions

- **Use this sheet as cover sheet and staple it to the assignment.** Write your name **legibly** in the space above; if necessary, underline your last name. If your name is not clearly and unambiguously identifiable on the class roster, we cannot credit you for the homework.
- Do the problems in order, and make sure that each problem is clearly labelled.
- Show all work; an answer alone will not earn credit.
- **Due date:** The assignment is due **in class** Friday this week; late homework, or homework dropped off in mailboxes, will not be accepted. You can, however, turn in the homework early, in my office, 241 Illini Hall, any time before the due date.
- **Open House Hours:** Wednesdays, 5 pm, 141 Altgeld. Backup slot: Thursdays, 5 pm, 141 Altgeld. I will stay as long as needed. The Open House is an informal office hour for students in my classes (Math 408 and 453). Feel free to stop by with questions about the homework or anything else relating to this course! Math 408 students should try to come to the Wednesday hour; on Thursday my Math 453 students will have priority, so use that slot only if you absolutely can't make it on Wednesday.

HW 6 Problems (from Hogg/Tanis, 7th edition)

Note on graphs: Several problems ask to graph the p.d.f. or c.d.f. of a random variable. Make sure that these graphs show the behavior of a p.d.f. or c.d.f., over the full range from $-\infty$ to ∞, i.e., including the behavior on the far right and on the far left of the x-axis (just like I did in the example in Friday's class with ranges labelled I, II, III, IV.

1. 3.2-1(a)(b)(c)
2. 3.2-3(c)
3. 3.2-5(c)
4. 3.2-10(c)
5. 3.2-11(c)
6. 3.2-14
7. 3.2-18
8. 3.2-21
9. 3.2-24
10. 3.2-25

***** Turn page for comments *****

Comments

The problems in this assignment all involve setting up an appropriate integral and then evaluating that integral. The set-up part is usually straightforward, but the integration part can present some pitfalls, and it's there where errors are typically made. Below are some tips and pointers to common sources of errors. (I pointed these out in class, but this bears repeating.)

- **If you have to compute a c.d.f. $F(x)$, or a p.d.f. $f(x)$, check if the functions you obtained satisfy the basic properties of such functions.** In particular: A c.d.f. $F(x)$ increases from 0 at the left end of the range (which may be $-\infty$) to 1 at the right end of the range; it may stay constant over intervals in between, but *a c.d.f. never decreases.* (A p.d.f., though, can increase and decrease.) A c.d.f. of a continuous random variable is necessarily continuous (i.e., can't have jumps); in particular, if it is defined by different formulas for different ranges, the values given by these formulas must match up at the boundary points of these ranges. (Note that a p.d.f. may have discontinuities.)

- **Always keep track of the "range" of a density function**. The "range" is the interval of x-values on which the function is defined and non-zero; it's the analog of the *list* of values for discrete random variables (i.e., the first row in a distribution table), which plays an equally important role for discrete random variables. **Make it a habit to write down the appropriate range along with the formula for the density**. Use this range in order to determine the correct integration limits. For example, if a problem asks for $P(X \geq 2)$ and the density $f(x)$ "lives" on the interval $[1, 4]$, the integral giving $P(X \geq 2)$ should be from 2 to 4; if, instead, $f(x)$ were defined and non-zero on the infinite interval $[1, \infty)$, the integral would be from2 to infinity.

- **Breaking up integrals:** If the definition of a density function $f(x)$ involves different formulas for different ranges, you'll need to break up integrals into separate integrals over these ranges. The same goes if the function involves an absolute value: for example, if the formula for $f(x)$ involves $|x|$, then you need to consider separately the range $x > 0$ (where $|x|$ can be replaced by x) and $x \leq 0$ (where $|x|$ can be replaced by $-x$). **You have to get rid of any absolute values before integrating.**

- **Techniques of integration:** You need to know integration by substitution and integration by parts. Both substitution and integration by parts are frequently needed for the typics of integrals that come up in actuarial exam problems. Typical examples are: $\int xe^{-x}dx$ (use integration by parts with $u = x, dv = e^{-x}dx$); $\int xe^{-x^2}dx$ (use substitution $u = x^2$, $du = 2xdx$). If you are rusty on these techniques, review the appropriate sections in your calculus text. (Note that trig and inverse trig integrals (such as $\int \sin^2 x \cos^2 x dx$, $\int \sqrt{1-x^2}dx$, $\int \sec x dx$, etc.), which occupy a big chunk of Calc II, are not important in probabilistic applications, so you need not know the various tricks associated with such integrals.)

- **Perform "sanity checks":** It is easy to make sign mistakes in integrals, but these are usually just as easy to catch because they lead to answers that don't pass basic "sanity checks". For example, the integral $\int_2^\infty e^{-2x}dx$ is equal to $(-1/2)e^{-2x}\Big|_2^\infty$, which is $(-1/2)(0 - e^{-4}) = (1/2)e^{-4}$, not $(-1/2)e^{-4}$. The latter answer, however, can easily be recognized as wrong since it's negative, and an integral over a positive function cannot evaluate to a negative number.