

Math 408, Spring 2008
HW Assignment 5, due Friday, 2/22/2008

Name (print please):

Instructions

- **Use this sheet as cover sheet and staple it to the assignment.** Write your name **legibly** in the space above; if necessary, underline your last name. If your name is not clearly and unambiguously identifiable on the class roster, we cannot credit you for the homework.
- Do the problems in order, and make sure that each problem is clearly labelled.
- Show all work; an answer alone will not earn credit.
- **Due date:** The assignment is due **in class** Friday this week; late homework, or homework dropped off in mailboxes, will not be accepted. You can, however, turn in the homework early, in my office, 241 Illini Hall, any time before the due date.
- **Open House Hours:** Wednesdays, 5 pm, 141 Altgeld. Backup slot: Thursdays, 5 pm, 141 Altgeld. I will stay as long as needed. The Open House is an informal office hour for students in my classes (Math 408 and 453). Feel free to stop by with questions about the homework or anything else relating to this course! Math 408 students should try to come to the Wednesday hour; on Thursday my Math 453 students will have priority, so use that slot only if you absolutely can't make it on Wednesday.

HW 5 Problems (from Hogg/Tanis, 7th edition)

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|----------------------------|--------------------------------------------------------------------|
| 1. 2.5-3 | 6. 2.6-4 |
| 2. 2.5-21(a)(b) (skip (c)) | 7. 2.6-10 (with factor $1/2$ instead of 2; see instructions below) |
| 3. 2.5-24(a)(b) | 8. 2.6-20 |
| 4. 2.6-1 | 9. 2.6-21 |
| 5. 2.6-2 | 10. 2.6-23 |

*** Turn page for instructions and comments ***

Comments

- **Problems 2.5-3 and 2.5-21:** These are problems on moment-generating functions; problems of this type have become more frequent in recent actuarial exams, so it's important to practice such problems. The assigned problems require the “reverse-engineering” of a moment-generating function: You are given an m.g.f., and you have to find the p.m.f. that produces the given m.g.f. (rather than the other way around, computing an m.g.f., given a p.m.f.). The trick here is to compare the given (concrete) formula for $M(t)$ with the general formula $M(t) = E(e^{tX}) = \sum e^{tx} f(x)$, and read off the appropriate values of x , as well as the corresponding probabilities $f(x)$; for example, a term $(1/2)e^{3t}$ in $M(t)$ means that $x = 3$ must be one of the values of X , and that the associated probability must be $f(3) = P(X = 3) = (1/2)$.

Once you have the p.m.f., you can compute probabilities such as $P(X \geq 1)$. (The expectation and variance can be obtained directly from the m.g.f. by differentiating, but for probabilities such as those above you need to compute the p.m.f. first.)

- **Problem 2.5-24(b):** This is just like the “world series problem” I did in class (involving the probability that the 4th success occurs at the 7th trial).
- **Problems from 2.6:** Most of these problems involve the Poisson distribution. As with the binomial distribution, **do not use the distribution tables for the Poisson distribution in the back of the book**; instead compute the Poisson probabilities directly via the p.m.f. You won't be able to use such a table in actuarial exams nor in any exams/quizzes in this class, so you have to get used to get by without. The assigned problems are such that you do not need a Poisson table to do them. (This is the reason I changed the numbers in 2.6-10; see below.)
- **Problem 2.6-10:** To reduce the calculations I changed the numbers a bit, replacing the factor 2 with $1/2$. **Thus, instead of $P(\mu - 2\sigma < X < \mu + 2\sigma)$ compute $P(\mu - (1/2)\sigma < X < \mu + (1/2)\sigma)$.**
- **Problem 2.6-21:** A problem just like (a) was done in class on 2/18. Part (b) is handled in much the same way as the expectation calculations in 2.2-1 or 2.2-2.
- **Problem 2.6-23:** The key here is that only 4 copies of the newspaper are available for sale. The number 4 is just like a “cap” (i.e., maximum payment) in insurance policies, and problems involving such caps are very important in actuarial applications and occur frequently in actuarial exams. For that reason, the problem is very instructional.

If an unlimited number of copies were available for sale, then the number of copies sold would be equal to the number of copies requested, which is given as a Poisson distribution random variable. Thus, to get the expected number of sold copies, one could use the formula for the expectation of a Poisson distributed random variable.

With the $n = 4$ cap in place, one has to deal with two random variables: the original Poisson-distributed variable, X , denoting number of copies **requested**, and another variable, say Y , denoting the number of copies **sold**. The problem then becomes much like Problems 2.2-1 or 2.2-2, and can be treated in a similar way: Draw up a distribution table of values x , with associated probabilities $f(x) = P(X = x)$ in the second row, and with associated values of Y in the third row. Then multiply each of the latter values with the corresponding probability and add everything up. This leads to an infinite series; which can be evaluated using the exponential series formula.