Math 408, Spring 2008
HW Assignment 3, due Friday, 2/8/2008

Name (print please):

Instructions

- **Use this sheet as cover sheet and staple it to the assignment.** Write your name **legibly** in the space above; if necessary, underline your last name. If your name is not clearly and unambiguously identifiable on the class roster, we cannot credit you for the homework.
- Do the problems in order, and make sure that each problem is clearly labelled.
- Show all work; an answer alone will not earn credit.
- **Due date:** The assignment is due **in class** Friday this week; late homework, or homework dropped off in mailboxes, will not be accepted. You can, however, turn in the homework early, in my office, 241 Illini Hall, any time before the due date.

HW 3 Problems (from Hogg/Tanis, 7th edition)

1. 2.1-3(a)(d)
2. 2.1-6(a)
3. 2.1-7(a)(c)
4. 2.1-8(a)
5. 2.2-1
6. 2.2-2
7. 2.2-4
8. 2.3-1(a)(b)(c)
9. 2.3-2(a)
10. 2.3-3(a)(b)(c)
11. 2.3-4

Comments

The problems in this set fall into two categories:

Type I: Explicit derivation of the p.m.f. for a given random variable.

Problems 6, 7, and 8 of 2.1 are exercises in *deriving* a p.m.f. for a given random variable. This is similar to Example 2.1-3 in the book (with X defined as the larger of the two numbers showing when rolling two four-sided dice). For these problems you have to (i) identify and list the outcomes (each occurring with the same probability), (ii) list the possible values x of X (iii) for each of these values describe (or depict by "bubbling in" the corresponding outcomes in the sample space, just like I did in class for the coin tossing example) the set of outcomes at which X is equal to this value, and (iv) use such a description/depiction to compute its probability (which, in the case of equally likely outcomes, boils down to counting elements in the "bubble" presenting the set).

The probabilities so computed are (by definition) the values of the p.m.f. $f(x)$, which is asked in the problems. Represent these values in the form of an explicit list ("$f(1) = 1/16$, $f(2) = 7/16$, etc.") or a distribution table, with x in the first row and $f(x)(= P(X = x))$ in the second.

The point of these problems is for you to *derive* each of the probabilities $P(X = x)$, and hence the p.m.f., $f(x)$. This is where the bulk of the work lies. **Just writing down the end result (a distribution table, or formula) won't earn you credit. You have to clearly show how you derived each of the table entries, e.g., by following the steps above.** (If the reasoning is the same for several entries, it suffices to do one explicitly; for the others you can simply say which pattern they follow.)

Important note: The book, in some examples (e.g., Ex. 2.1-3) and in the answers to some problems (e.g., Problem 2.1-7) gives $f(x)$ by a simple looking formula (such as $f(x) = (2x-1)/16$). *This is a bad and highly misleading practice*, as it suggests that the formula for $f(x)$ can be obtained wholesale in a single step. However, this is rarely the case: in nearly all of these problems, $f(x)$ has to be computed *one value at a time.* For example, if x can take four values $1, 2, 3, 4$, then four such computations are needed, one for each of these four values. This is exactly what the book does in Example 2.1-3 (see the bottom of page 60), computing $f(1) = P(X = 1) = 1/16$, $f(2) = 3/16$, $f(3) = 5/16$, and $f(4) = 7/16$. It is only *after having computed these values one at a time* that the book goes on to say that $f(x)$ "can be written simply as $f(x) = (2x - 1)/16$". The latter observation, while correct, seems like pulled out of the air (would you have come up with this formula on your own, after seeing the four values 1/16, 3/16, 5/16, 7/16???), it serves no useful purpose, and, worst of all, it misleads one into thinking that one can obtain the answer in the formula version in one simple step through some algebraic magic.

Bottomline: *In problems of this type, specify the values of $f(x)$ as an explicit list (e.g., "$f(1) = 1/16$, $f(2) = 7/16$, etc.") or in the form of a distribution table, with the x's in the first row, and the corresponding values $f(x)$ in the second, but don't try to fit the result into a formula.*

Checking p.m.f.'s: Though the problems do not explicitly ask for this, I highly recommend that, in addition, you check that the probabilities (i.e., the entries in the second row of a distribution table) add up to 1; if they don't, you know you have made a mistake.

Histograms and bar graphs: A few of the problems ask to represent the p.m.f.'s obtained as histograms or bar graphs. While you should be familiar with these concepts (which are very simple and intuitive), you can ignore those questions. The main thing is that you compute the p.m.f.'s correctly.

Type II: Application of definitions, rules, and properties:

The remaining problems (2.1-3 and the problems in 2.2 and 2.3) are abstract exercises in applying definitions, rules, and properties of densities, expectations, etc. All you need for those problems are the formulas on the first page of the Discrete Random Variables handout.

Problems 2.2-1 and 2.2-2 are also of the this type, though disguised as word problems. (In fact, those two problems were taken from past Actuarial Exams). This may make them *seem* harder, though once the verbal disguise is stripped away, the underlying mathematical problem is not particularly hard. For both problems it is useful to represent $f(x)$ by a distribution table, and add a third row denoting the payment corresponding to each of the x-values.

Problem 2.3-4 seems more abstract and daunting, but it, too, is nothing more than a repeated application of the basic rules for expectation ($E(cX) = cE(X)$, $E(X+Y) = E(X)+E(Y)$, etc.), and the definitions of $\mu(= E(X))$ and $\sigma^2 = E(X^2) - E(X)^2$. Keep in mind that μ and σ^2 are constants, and you can work with these quantities as if they were concrete numbers. After applying the rules, simplifying, and substituting the definitions of μ and σ^2, it turns out that almost everything cancels or drops out, and the final answers are very simple.