Math 408, Actuarial Statistics I, Spring 2008

Combinatorial problems

Word counting problems

1. Find the number of possible 10 character passwords under the following restrictions: (Note there are 26 letters in the alphabet.)
   (a) All characters must be lower case letters.
   (b) All characters must be lower case letters and distinct.
   (c) Letters and digits must alternate and be distinct (as in "1w2x9c4u5s" or "a1b2c3d4e5").
   (d) All characters must be lower case, distinct, and *in alphabetical order*. (e.g., "abfghikmno" is allowed, but not "bafghikmno").
   (e) The word can only contain the upper case letters A and B.
   (f) The word can only contain the upper case letters A and B, and must contain each of these letters.
   (g) The word can only contain the upper case letters A and B, and must contain an equal number of each.
2. Find the number of different words that can be formed by rearranging the letters in the following words: (Include the given word in the count.)
   (a) NORMAL
   (b) HHTTTT
   (c) ILLINI
   (d) MISSISSIPPI
3. How many ways are there to seat 10 people, consisting of 5 couples, in a row of seats (10 seats wide) if
   (a) the seats are assigned at random?
   (b) all couples are to get adjacent seats?
4. Assume the final exams are given over a period of 6 days, with 3 slots per day, so that there are a total of 18 final exam slots. If you have 5 classes, each with a final exam, what is the probability that your 5 finals fall onto different days, assuming that the 5 classes all have different final exam slots?

Birthday type problems

1. What is the probability that in a group of n no two people share a common birthday? Work out the probabilities numerically for $n = 20, 30, 40, 50, 60$. (Assume there are 365 days in a year.)

2. Six people get into an elevator at the ground floor of a hotel which has 10 upper floors. Assuming each person gets off at a randomly chosen floor, what is the probability that no two people get off at the same floor?
3. Suppose you record the birthdays of a large group of people, one at a time (just like in the birthday simulation posted on the course webpage) until you have found a match, i.e., a birthday that has already been recorded.
   (a) What is the probability that it takes *more than* 20 people for this to occur?
   (b) What is the probability that it takes *exactly* 20 people for this to occur?
4. Suppose you record the birthdays of a large group of people, one at a time, until you have found a person whose birthday *matches your own birthday.* What is the probability that it takes exactly 20 people for this to occur?

Urn/ball type problems

1. **The classical urn/ball problem.** An urn (box) contains 30 balls, of which 10 are red and the other 20 blue. Suppose you take out 8 balls from this urn, without replacement. What is the probability that among the 8 balls in this sample exactly 3 are red and 5 are blue?
2. **The committee problem.** Assume a committee of 10 has to be selected from a group of 100 people, of which 40 are men and 60 are women.
   (a) How many ways are there to choose such a committee?
   (b) How many ways are there to choose the committee so that exactly half of the members are men?
   (c) What is the probability that a randomly selected committee of 10 consists of exactly 5 men and 5 women?
3. **The lottery problem.** In a state lottery a player has to choose 6 (distinct) numbers out of 54 numbers. At each drawing, 6 numbers are drawn at random, without replacement, from these 54 numbers. You win a first prize if all 6 drawn numbers agree with the ones you picked, a second prize if exactly 5 of the drawn numbers agree, etc. Compute the probabilities of winning a first prize, a second prize, etc.

Additional examples/problems from Hogg/Tanis

1. Example 1.3-9:
2. Example 1.3-10
3. Example 1.3-11
4. Example 1.3-12
5. Problem 1.3-1
6. Problem 1.3-3
7. Problem 1.3-5
8. Problem 1.3-7
9. Problem 1.3-11