Math 370, Actuarial Problemsolving
Spring 2008
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Practice Test, 1/28/2008 (with solutions)

About this test. This is a practice test made up of a random collection of 20 problems from past Course 1/P actuarial exams. Most of the problems have appeared on the Actuarial Problem sets passed out in class, but I have also included a number of additional problems.

Topics covered. This test covers the topics of the entire actuarial exam syllubus, corresponding to Chapters 1–5 in Hogg/Tanis and Actuarial Problem Sets 1–5: General probability, discrete distributions, continuous distributions, multivariate distributions, and normal approximation and the Central Limit Theorem.

Ordering of the problems. In order to mimick the conditions of the actual exam as closely as possible, the problems are in no particular order. Easy problems are mixed in with hard ones. In fact, I used a program to select the problems and to put them in random order, with no human intervention.

Rules. This test is intended to simulate the real thing as closely as possible, so you should abide by the same rules. In particular, no notes, books, etc., and use calculators only for basic arithmetic operations. **In particular, do not use calculators to compute integrals, derivatives, or to plot graphs.** In the actuarial exam you are limited to calculators without such functions. It goes without saying that you shouldn't cheat. Don't copy an answer from your neighbor; if you do so, you are only cheating yourself, and you are defeating the purpose of this course.

Time. You have 2:00 hours for this exam. This works out to 6 minutes per problem, which is the same as what you get in an actuarial exam. Use the time wisely, and don't let yourself get bogged down in a lengthy calculation. If you don't get a problem at first, move on to the next one.

Answers/solutions. I will post an answer key and partial solutions on the course webpage, `www.math.uiuc.edu/~hildebr/370`.

1. [Problem 1-5]

   An actuary studying the insurance preferences of automobile owners makes the following conclusions:

   (i) An automobile owner is twice as likely to purchase collision coverage as disability coverage.

   (ii) The event that an automobile owner purchases collision coverage is independent of the event that he or she purchases disability coverage.

   (iii) The probability that an automobile owner purchases both collision and disability coverages is 0.15.

   What is the probability that an automobile owner purchases neither collision nor disability coverage?

   (A) 0.18 (B) 0.33 (C) 0.48 (D) 0.67 (E) 0.82

   Answer: B: 0.33

   Source: Problem 30/nov00

   Hint/Solution: Let C denote collision coverage, and D disability coverage. We are given that (i) $P(C) = 2P(D)$, (ii) C and D are independent, and (iii) $P(C \cap D) = 0.15$, and we need to compute $P(C' \cap D')$.

   By independence, $P(C \cap D) = P(C)P(D)$. Using the independence along with the equations (i) and (iii) we get $0.15 = P(C \cap D) = P(C)P(D) = 2P(D)^2$, so $P(D) =$ and $P(C) =$. Hence $P(C') =$ and $P(D') =$. Applying indendence again, we get $P(C' \cap D') = P(C')P(D')$.

2. [Problem 4-103]

   A device runs until either of two components fails, at which point the device stops running. The joint density function of the lifetimes of the two components, both measured in hours, is

   $$f(x, y) = \begin{cases} \frac{1}{27}(x + y) & \text{for } 0 < x < 3 \text{ and } 0 < y < 3, \\ 0 & \text{otherwise.} \end{cases}$$

   Calculate the probability that the device fails during its first hour of operation.

   (A) 0.04 (B) 0.41 (C) 0.44 (D) 0.59 (E) 0.96

   Answer: B: 0.41

   Source: Problem 16/may03

   Hint/Solution: The tricky part here is to properly interpret the statement "the device fails during first hour", and identify the corresponding region in the xy-plane; once this is done, the problem amounts to computing a relatively simple double integral.

3. [Problem 1-8]

   An insurance company pays hospital claims. The number of claims that include emergency room or operating room charges is 85% of the total number of claims. The number of claims that do not include emergency room charges is 25% of the total number of claims. The occurrence of emergency room charges is independent of the occurrence of operating room charges on hospital claims. Calculate the probability that a claim submitted to the insurance company includes operating room charges.

   (A) 0.10 (B) 0.20 (C) 0.25 (D) 0.40 (E) 0.80

   Answer: D: 0.40

   Source: Problem 37/may03

   Hint/Solution: The relevant events are "operating room charges" and "emergency room charges"; we are given the probability of the union of these two events, and the probability of the complement of the second one, and we need to compute the probability of the first; for that, either use appropriate probability rules (e.g., the formula for the probability of a union), or determine the probability from a Venn diagram.

4. [Problem 3-24]

   The value, ν, of an appliance is given by $\nu(t) = e^{7-0.2t}$, where t denotes the number of years since purchase. If appliance fails within seven years of purchase, a warranty pays the owner the value of the appliance at the time of failure. After seven years, the warranty pays nothing. The time until failure of the appliance has an exponential distribution with mean 10. Calculate the expected payment from the warranty.

   (A) 98.70 (B) 109.66 (C) 270.43 (D) 320.78 (E) 352.16

   Answer: D: 320.78

   Source: Problem 23/1999

   Hint/Solution: Let X denote the payout under the warranty, and T the time until failure. Then $X = e^{7-0.2T}$ if $T \leq 7$, and $X = 0$ otherwise. Since T has exponential density with mean 10, its density function is $(1/10)e^{-t/10}$ for $t > 0$. Thus,

$$\begin{aligned} E(X) &= \int_0^7 e^{7-0.2t}\frac{1}{10}e^{-t/10}dt \\ &= 0.1 \cdot e^7 \int_0^7 e^{-0.3t}dt == 0.1 \cdot e^7 \frac{e^{-0.3\cdot 7} - 1}{-0.3} = 320.78 \end{aligned}$$

5. [Problem 3-11]

   An insurance policy reimburses a loss up to a benefit limit of 10. The policyholder's loss, Y, follows a distribution with density function

$$f(x) = \begin{cases} 2y^{-3} & \text{for } y > 1, \\ 0 & \text{otherwise.} \end{cases}$$

What is the expected value of the benefit paid under the insurance policy?

(A) 1.0 (B) 1.3 (C) 1.8 (D) 1.9 (E) 2.0

Answer: D: 1.9

Source: Problem 34/may00

Hint/Solution: The benefit, X, is given by $X = Y$ if $Y \le 10$, and by $X = 10$ if $Y > 10$. Thus,

$$E(X) = \int_1^{10} y \cdot 2y^{-3} dy + \int_{10}^{\infty} 10 \cdot 2y^{-3} dy = 2\left(1 - \frac{1}{10}\right) + 10 \cdot 10^{-2} = 1.9$$

6. [Problem 4-15]

A company offers a basic life insurance policy to its employees, as well as a supplemental life insurance policy. To purchase the supplemental policy, an employee must first purchase the basic policy. Let X denote the proportion of employees who purchase the basic policy, and Y the proportion of employees who purchase the supplemental policy. Let X and Y have the joint density function $f(x, y) = 2(x + y)$ on the region where the density is positive. Given that 10% of the employees buy the basic policy, what is the probability that fewer than 5% buy the supplemental policy?

(A) 0.010 (B) 0.013 (C) 0.108 (D) 0.417 (E) 0.500

Answer: D: 0.417

Source: Problem 11/may00

Hint/Solution: We need to compute the conditional probability $P(Y \le 0.05|X = 0.1)$. This is given by $\int_{y=0}^{0.05} h(y|0.1)dy$, where $h(y|0.1)$ is the conditional density of Y given that $X = 0.1$. Now, $h(y|0.1) = f(0.1, y)/f_X(0.1)$, where $f(x, y) = 2(x + y)$ is the given density and $f_X(x)$ is the marginal density of X. The main difficulty of the problem is the proper interpretation of the phrase "on the region where the joint density is positive". Obviously, since x and y both denote proportions, this region must be part of the unit square $0 \le x \le 1, 0 \le y \le 1$. However, an additional restriction is imposed on x and y by the requirement that to purchase a supplemental policy, one must first purchase the basic policy. This means that the proportion of supplemental policy holders cannot be larger than the proportion of basic policy holders. Thus, x and y are restricted by the additional condition $y \le x$, and the region in question is the triangular region given by $0 \le x \le 1, 0 \le y \le x$. Using these limits on x and y in the integrations, we then get

$$f_X(x) = \int_{y=0}^{y=x} (2x + y)dy = 3x^2 \quad (0 \le x \le 1),$$

so $f_X(0.1) = 0.03$ and $h(y|0.1) = 2(0.1 + y)/0.03 = (200/3)(0.1 + y)$ for $0 \le y \le 0.05$. Finally, substituting this into the above formula for $P(Y \le 0.05|X = 0.1)$ gives

$$P(Y \le 0.05|X = 0.1) = \frac{200}{3} \int_0^{0.05} (0.1 + y)dy = \frac{200}{3}\left(0.05 \cdot 0.1 + \frac{0.05^2}{2}\right) = 0.4166.$$

7. [Problem 4-54]

An auto insurance policy will pay for damage to both the policyholder's car and the other driver's car in the event that the policyholder is responsible for an accident. The size of the payment for damage to the policyholder's car, X, has a marginal density function of 1 for $0 < x < 1$. Given $X = x$, the size of the payment for damage to the other driver's car, Y, has conditional density of 1 for $x < y < x + 1$. If the policyholder is responsible for an accident, what is the probability that the payment for damage to the other driver's car will be greater than 0.500?

(A) 3/8 (B) 1/2 (C) 3/4 (D) 7/8 (E) 15/16

Answer: D: 7/8

Source: Problem 34/nov01

Hint/Solution: The probability to compute is $P(Y \geq 0.5)$. We are given that X is uniform on $[0, 1]$ and that, given $X = x$, Y is uniform on $[x, x + 1]$. Thus, $f_X(x) = 1$ for $0 \leq x \leq 1$ and $h(y|x) = 1$ for $x \leq y \leq x + 1$, $0 \leq x \leq 1$. Hence

$$f(x, y) = f_X(x)h(y|x) = 1, \quad 0 \leq x \leq 1, x \leq y \leq x + 1.$$

In particular, this means that the joint density is uniform on the region (a parallelogram—sketch!) $0 \leq x \leq 1$, $x \leq y \leq x + 1$. Hence probabilities can be computed as ratios of areas. The entire parallelogram has area 1, and the part of this parallelogram on which $y \geq 0.5$ has area $1 - (1/2)0.5^2 = 7/8$ by elementary trigonometry. Hence $P(Y \geq 0.5) = (7/8)/1 = 7/8$.

8. [Problem 3-20]

A device that continuously measures and records seismic activity is placed in a remote region. The time, T, to failure of this device is exponentially distributed with mean 3 years. Since the device will not be monitored during its first two years of service, the time to discovery of its failure is $X = \max(T, 2)$. Determine $E(X)$.

(A) $2 + \frac{1}{3}e^{-6}$
(B) $2 - 2e^{-2/3} + 5e^{-4/3}$
(C) 3
(D) $2 + 3e^{-2/3}$
(E) 5

Answer: D

Source: Problem 20/may01

Hint/Solution: Note that $X = T$ if $T \geq 2$ and $X = 2$ if $0 \leq T < 2$, Since T is exponentially distributed with mean 3, the density function of T is $f(t) = (1/3)e^{-t/3}$

for $t > 0$. Thus,

$$\begin{aligned} E(X) = E(\max(T,2)) &== \int_0^\infty \max(t,2)(1/3)e^{-t/3}dt \\ &= \int_0^2 \frac{2}{3}e^{-t/3}dt + \int_2^\infty \frac{t}{3}e^{-t/3}dt \\ &= 2(1-e^{-2/3}) - te^{-t/3}\Big|_2^\infty + \int_2^\infty e^{-t/3}dt \\ &= 2(1-e^{-2/3}) + 2e^{-t/3} + 3e^{-2/3} = 2 + 3e^{-2/3} \end{aligned}$$

9. [Problem 3-13]

The time, T, that a manufacturing system is out of operation has cumulative distribution function

$$F(t) = \begin{cases} 1 - \left(\frac{2}{t}\right)^2 & \text{for } t > 2, \\ 0 & \text{otherwise.} \end{cases}$$

The resulting cost to the company is $Y = T^2$. Determine the density function of Y, for $y > 4$.

(A) $4y^{-2}$ (B) $8y^{-3/2}$ (C) $8y^{-3}$ (D) $16y^{-1}$ (E) $1024y^{-5}$

Answer: A

Source: Problem 23/may03

Hint/Solution: This is a routine application of the change of variables techniques; as always in these problems, you need to take a detour via the corresponding c.d.f.'s to get the p.d.f. of the new variable.

10. [Problem 5-51]

Let X and Y be the number of hours that a randomly selected person watches movies and sporting events, respectively, during a three-month period. The following information is known about X and Y:

$$E(X) = 50, \quad E(Y) = 20, \quad \mathrm{Var}(X) = 50, \quad \mathrm{Var}(Y) = 30, \quad \mathrm{Cov}(X,Y) = 10.$$

One hundred people are randomly selected and observed for these three months. Let T be the total number of hours that these one hundred people watch movies or sporting events during this three-month period. Approximate the value of $P(T < 7100)$.

(A) 0.62 (B) 0.84 (C) 0.87 (D) 0.92 (E) 0.97

Answer: B: 0.84

Source: Problem 40/nov01

Hint/Solution: The total time that a *single* person spends watching is a random variable with the distribution of $X + Y$ and mean and variance given by

$$\begin{aligned} \mu &= E(X+Y) = E(X) + E(Y) = 50 + 20 = 70, \\ \sigma^2 &= \mathrm{Var}(X+Y) = \mathrm{Var}(X) + \mathrm{Var}(Y) + 2\,\mathrm{Cov}(X,Y) = 50 + 30 + 2\cdot 10 = 100. \end{aligned}$$

The total time, T, spent by 100 such people is a sum of 100 independent random variables, each of the type $X+Y$. By the CLT, T has approximately normal distribution with mean $100\mu = 100\cdot 70 = 7000$ and standard deviation $\sqrt{100}\sigma = \sqrt{100}\cdot\sqrt{100} = 100$. Hence

$$P(T < 7100) = P\left(\frac{T-7000}{100} < \frac{7100-7000}{100}\right) \approx P(Z < 1) = \Phi(1) = 0.84.$$

11. [Problem 4-51]

Once a fire is reported to a fire insurance company, the company makes an initial estimate, X, of the amount it will pay to the claimant for the fire loss. When the claim is finally settled, the company pays an amount, Y, to the claimant. The company has determined that X and Y have the joint density function

$$f(x,y) = \frac{2}{x^2(x-1)} y^{-(2x-1)/(x-1)}, \quad x > 1, y > 1.$$

Given that the initial claim estimated by the company is 2, determine the probability that the final settlement amount is between 1 and 3.

(A) 1/9 (B) 2/9 (C) 1/3 (D) 2/3 (E) 8/9

Answer: E: 8/9

Source: Problem 25/nov01

Hint/Solution: We need to compute $P(1 \le Y \le 3|X=2)$. This is given by the integral $(*)$ $\int_1^3 h(y|2)$, where $h(y|2)$ is the conditional density of Y given $X=2$. Now, $h(y|2) = f(2,y)/f_X(2)$, where $f(2,y) = (1/2)y^{-3}$ from the given joint density and $f_X(2)$ is the marginal density of X, at $x=2$, i.e.,

$$f_X(2) = \int_{y=1}^{\infty} f(2,y)dy = \int_{y=1}^{\infty} \frac{1}{2} y^{-3} dy = \frac{1}{4}.$$

Thus, $h(y|2) = 2y^{-3}$ for $1 < y < \infty$. Substituting this density into $(*)$ gives the answer:

$$P(1 \le Y \le 3|X=2) = \int_1^3 2y^{-3} dy = \frac{8}{9}.$$

12. [Problem 4-53]

A company is reviewing tornado damage claims under a farm insurance policy. Let X be the portion of a claim representing damage to the house and let Y be the portion of the same claim representing damage to the rest of the property. The joint density function of X and Y is

$$f(x,y) = \begin{cases} 6(1-(x+y)) & \text{for } x > 0, y > 0, x+y < 1, \\ 0 & \text{otherwise.} \end{cases}$$

Determine the probability that the portion of a claim representing damage to the house is less than 0.2.

(A) 0.360 (B) 0.480 (C) 0.488 (D) 0.512 (E) 0.520

Answer: C: 0.488

Source: Problem 5/may01

Hint/Solution: The probability to compute is $P(X \le 0.2)$, which can be computed as the integral over the joint density $f(x,y)$ over the part of its range where $x \ge 0.2$. Now the range of $f(x,y)$ is the triangle given by $0 \le x < 1$, $0 \le y < 1-x$ (sketch!), and restricting x further by $0 \le x \le 0.2$, we get the desired range of integration:

$$\begin{aligned} P(X \le 0.2) &= \int_{x=0}^{0.2} \int_{y=0}^{1-x} 6(1-(x+y))dxdy \\ &= \int_{x=0}^{0.2} 3(1-x)^2 dx = 1 - 0.8^3 = 0.488 \end{aligned}$$

13. [Problem 2-8]

The distribution of loss due to fire damage to a warehouse is:

Amount of loss	Probability
0	0.900
500	0.060
1,000	0.030
10,000	0.008
50,000	0.001
100,000	0.001

Given that a loss is greater than zero, calculate the expected amount of the loss.

(A) 290 (B) 322 (C) $1,704$ (D) $2,900$ (E) $32,222$

Answer: D: 2,900

Source: Problem 9/1999

Hint/Solution: This would be a straightforward calculation except for the phrase "given that the loss is greater than 0". The correct interpretation of this condition is to restrict consideration to those cases where the loss is greater than 0 and renormalize the probabilities for these cases so that they again add up to 1. Since the probability for a loss of 0 is 0.9, the remaining probabilities must add up to 0.1, so dividing each by 0.1 renormalizes them to have sum 1. Using these renormalized probabilities in the formula for an expectation gives the desired expectation under the above condition.

14. [Problem 1-104]

    An auto insurance company has $10,000$ policyholders. Each policyholder is classified as

    (i) young or old;
    (ii) male or female; and
    (iii) married or single.

    Of these policyholders, 3000 are young, 4600 are male, and 7000 are married. The policyholders can also be classified as 1320 young males, 3010 married males, and 1400 young married persons. Finally, 600 of the policyholders are young married males. How many of the company's policyholders are young, female, and single?

    (A) 280 (B) 423 (C) 486 (D) 880 (E) 896

    Answer: D: 880

    Source: Problem 3/nov00

    Hint/Solution: If we consider the sets "young", "male" and "married", we want to count the part of the "young" set that is outside the other two sets. This is easy by drawing a Venn diagram and using the given data.

15. [Problem 4-56]

    Two insurers provide bids on an insurance policy to a large company. The bids must be between 2000 and 2200. The company decides to accept the lower bid if the two bids differ by 20 or more. Otherwise, the company will consider the two bids further. Assume that the two bids are independent and are both uniformly distributed on the interval from 2000 to 2200. Determine the probability that the company considers the two bids further.

    (A) 0.10 (B) 0.19 (C) 0.20 (D) 0.41 (E) 0.60

    Answer: B: 0.19

    Source: Problem 28/nov01

    Hint/Solution: We need to compute $P(|X - Y| < 20)$, where X and Y denote the two bids. We are given that X and Y are independent, with each having uniform distribution on $[2000, 2200]$. Thus, the joint distribution of X and Y is uniform on the square $[2000, 2200] \times [2000, 2200]$. The event $|X - Y| < 20$ corresponds to a diagonal strip inside this square. Since the joint distribution is uniform over the whole square, the probability of this event is given by the ratio between the area of this strip and the area of the entire square. A simple geometric argument (sketch!) shows that the strip has area $200^2 - 180^2$, so $P(|X - Y| < 20) = (200^2 - 180^2)/200^2 = 0.19$.

16. [Problem 3-15]

The lifetime of a printer costing 200 is exponentially distributed with mean 2 years. The manufacturer agrees to pay a full refund to a buyer if the printer fails during the first year following its purchase, and a one-half refund if it fails during the second year. If the manufacturer sells 100 printers, how much should it expect to pay in refunds?

(A) $6,321$ (B) $7,358$ (C) $7,869$ (D) $10,256$ (E) $12,642$

Answer: D: 10,256

Source: Problem 3/may00

Hint/Solution: Let X denote the lifetime of the printer. Then the refund, Y, is given by $Y = 200$ if $X \le 1$, $Y = 100$ if $1 \le X \le 2$, and $Y = 0$ otherwise. Thus, $E(Y) = 200P(X \le 1) + 100P(1 < X \le 2)$. The probabilities $P(\dots)$ here are easily computed using that fact X has exponential distribution with mean 2, and thus c.d.f. $F(x) = P(X \le x) = 1 - e^{-x/2}$ for $x \ge 0$. Thus, $P(X \le 1) = 1 - e^{-1/2}$, $P(1 < X \le 2) = P(X \le 2) - P(X \le 1) = (1 - e^{-2/2}) - (1 - e^{-1/2}) = e^{-1/2} - e^{-1}$, and hence $E(Y) = 200 \cdot +100 \cdot = 10,256$.

17. [Problem 3-9]

The monthly profit of Company I can be modeled by a continuous random variable with density function f. Company II has a monthly profit that is twice that of Company I. Determine the probability density function of the monthly profit of Company II.

(A) $\frac{1}{2}f\left(\frac{x}{2}\right)$ (B) $f\left(\frac{x}{2}\right)$ (C) $2f\left(\frac{x}{2}\right)$ (D) $2f(x)$ (E) $2f(2x)$

Answer: A

Source: Problem 32/nov00

Hint/Solution: This is an easy exercise in changing variables in density functions if you apply to correct procedure for that. Let X denote the profit for Company I and Y that for Company II. Then $Y = 2X$. Let $f(x)$ and $F(x)$ denote the p.d.f. and c.d.f. of X, and let $g(x)$ and $G(x)$ denote the corresponding functions for Y. Then we have $G(x) = P(Y \le x) = P(2X \le x) = P(X \le x/2) = F(x/2)$. Differentiating both sides with respect to x we get $g(x) = G'(x) = (1/2)F'(x/2) = (1/2)f(x/2)$ since $F'(x) = f(x)$. Thus, the density of Y is $(1/2)f(x/2)$.

18. [Problem 4-55]

An insurance policy is written to cover a loss X where X has density function

$$f(x) = \begin{cases} \frac{3}{8}x^2 & \text{for } 0 \le x \le 2, \\ 0 & \text{otherwise.} \end{cases}$$

The time (in hours) to process a claim of size x, where $0 \le x \le 2$, is uniformly distributed on the interval from x to $2x$. Calculate the probability that a randomly chosen claim on this policy is processed in three hours or more.

(A) 0.17 (B) 0.25 (C) 0.32 (D) 0.58 (E) 0.83

Answer: A: 0.17

Source: Problem 23/may00

Hint/Solution: We need to compute $P(Y \geq 3)$, where Y denotes the time needed to process a claim. From the problem, we know that, given $X = x$, Y is uniformly distributed on $[x, 2x]$. Thus, the conditional density of Y given $X = x$ is $h(y|x) = 1/x$ for $x \leq y \leq 2x$. Using this density along with the given (ordinary) density of X, we can compute the joint density of X and Y:

$$f(x,y) = h(y|x)f_X(x) = \frac{1}{x} \cdot \frac{3}{8}x^2 = \frac{3}{8}x, \quad 0 \leq x \leq 2, x \leq y \leq 2x.$$

A careful sketch of the region given here shows that the part of it that satisfies $y \geq 3$ is described by the inequalities $3/2 \leq x \leq 2$, $3 \leq y \leq 2x$. Hence,

$$P(Y \geq 3) = \int_{x=3/2}^{2} \int_{y=3}^{2x} 38xdydx = \int_{x=3/2}^{2} \frac{3}{8}x(2x-3)dx.$$

and calculating the latter integral we get the answer 0.17.

19. [Problem 5-3]

In an analysis of healthcare data, ages have been rounded to the nearest multiple of 5 years. The difference between the true age and the rounded age is assumed to be uniformly distributed on the interval from -2.5 years to 2.5 years. The healthcare data are based on a random sample of 48 people. What is the approximate probability that the mean of the rounded ages is within 0.25 years of the mean of the true ages?

(A) 0.14 (B) 0.38 (C) 0.57 (D) 0.77 (E) 0.88

Answer: D: 0.77

Source: Problem 19/may00

Hint/Solution: We need to compute $P(|\overline{X}| \leq 0.25)$, where $\overline{X}$ is the mean of a random sample of size 48 from a uniform distribution on $[-2.5, 2.5]$. Obviously this distribution has mean 0, and an easy computation (or using the formula $\sigma^2 = (b-a)^2/12$ for the variance of a uniform distribution on an interval $[a, b]$) shows that the standard deviation is 1.44. By the CLT, $\overline{X}$ has approximately normal distribution with mean 0 and standard deviation $1.44/\sqrt{48} = 0.2078$, so

$$\begin{aligned} P(|\overline{X}| \leq 0.25) &= P\left(\left|\frac{\overline{X}}{0.2078}\right| \leq \frac{0.25}{0.2078}\right) \\ &\approx P(|Z| \leq 1.2) = \Phi(1.2) - \Phi(-1.2) = 0.769. \end{aligned}$$

20. [Problem 3-7]

The time to failure of a component in an electronic device has an exponential distribution with a median of four hours. Calculate the probability that the component will work without failing for at least five hours.

(A) 0.07 (B) 0.29 (C) 0.38 (D) 0.42 (E) 0.57

Answer: D: 0.42

Source: Problem 4/may03

Hint/Solution: The general form of the c.d.f. of an exponential distribution is $F(x) = 1 - e^{-x/\theta}$ for $x > 0$. Since the median is 4 hours, we have $F(4) = 1/2$. This allows us to determine the parameter θ: $1 - e^{-4/\theta} = 1/2$, so $\theta = 4/\ln 2$. The probability that the component will work for at least 5 hours is given by $1 - F(5) = e^{-5/\theta} = e^{-(5/4)\ln 2} = 0.42$.