Math 370, Actuarial Problemsolving
Spring 2008
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Practice Test, 1/28/2008

About this test. This is a diagnostic test made up of a random collection of 20 problems from past Course 1/P actuarial exams. It is intended for students who have already taken of Math 408 or equivalent (e.g., Math 461), to give them an idea of where they stand. It also helps me identify areas that should be emphasized in this course. **This test will not count, so if you do poorly, it won't affect your grade. If you want, you can take the test anonymously—just leave out your name on the answer sheet. However, in order for the test to provide meaningful feedback, I'd like everyone who is taking the test to turn in an answer sheet.**

Note for those currently enrolled in 408: If you have not already taken 408, it does not make much sense to take this test, since most of the problems are on material not yet covered in 408. Instead work the set-theory practice problems.

Rules. This test is intended to simulate the real thing as closely as possible, so you should abide by the same rules. In particular, no notes, books, etc., and use calculators only for basic arithmetic operations. **In particular, do not use calculators to compute integrals, derivatives, or to plot graphs.** In the actuarial exam you are limited to calculators without such functions. It goes without saying that you shouldn't cheat. Don't copy an answer from your neighbor; if you do so, you are only cheating yourself, and you are defeating the purpose of this course.

Time. You have 2:00 hours for this exam. This works out to 6 minutes per problem, which is the same as what you get in an actuarial exam. Use the time wisely, and don't let yourself get bogged down in a lengthy calculation. If you don't get a problem at first, move on to the next one.

Answers/solutions. I will post an answer key and partial solutions on the course webpage, `www.math.uiuc.edu/~hildebr/370`.

1. An actuary studying the insurance preferences of automobile owners makes the following conclusions:

   (i) An automobile owner is twice as likely to purchase collision coverage as disability coverage.

   (ii) The event that an automobile owner purchases collision coverage is independent of the event that he or she purchases disability coverage.

   (iii) The probability that an automobile owner purchases both collision and disability coverages is 0.15.

   What is the probability that an automobile owner purchases neither collision nor disability coverage?

   (A) 0.18 (B) 0.33 (C) 0.48 (D) 0.67 (E) 0.82

2. A device runs until either of two components fails, at which point the device stops running. The joint density function of the lifetimes of the two components, both measured in hours, is

$$f(x,y) = \begin{cases} \frac{1}{27}(x+y) & \text{for } 0 < x < 3 \text{ and } 0 < y < 3, \\ 0 & \text{otherwise.} \end{cases}$$

Calculate the probability that the device fails during its first hour of operation.

(A) 0.04 (B) 0.41 (C) 0.44 (D) 0.59 (E) 0.96

3. An insurance company pays hospital claims. The number of claims that include emergency room or operating room charges is 85% of the total number of claims. The number of claims that do not include emergency room charges is 25% of the total number of claims. The occurrence of emergency room charges is independent of the occurrence of operating room charges on hospital claims. Calculate the probability that a claim submitted to the insurance company includes operating room charges.

   (A) 0.10 (B) 0.20 (C) 0.25 (D) 0.40 (E) 0.80

4. The value, ν, of an appliance is given by $\nu(t) = e^{7-0.2t}$, where t denotes the number of years since purchase. If appliance fails within seven years of purchase, a warranty pays the owner the value of the appliance at the time of failure. After seven years, the warranty pays nothing. The time until failure of the appliance has an exponential distribution with mean 10. Calculate the expected payment from the warranty.

   (A) 98.70 (B) 109.66 (C) 270.43 (D) 320.78 (E) 352.16

5. An insurance policy reimburses a loss up to a benefit limit of 10. The policyholder's loss, Y, follows a distribution with density function

$$f(x) = \begin{cases} 2y^{-3} & \text{for } y > 1, \\ 0 & \text{otherwise.} \end{cases}$$

What is the expected value of the benefit paid under the insurance policy?

(A) 1.0 (B) 1.3 (C) 1.8 (D) 1.9 (E) 2.0

6. A company offers a basic life insurance policy to its employees, as well as a supplemental life insurance policy. To purchase the supplemental policy, an employee must first purchase the basic policy. Let X denote the proportion of employees who purchase the basic policy, and Y the proportion of employees who purchase the supplemental policy. Let X and Y have the joint density function $f(x, y) = 2(x + y)$ on the region where the density is positive. Given that 10% of the employees buy the basic policy, what is the probability that fewer than 5% buy the supplemental policy?

   (A) 0.010 (B) 0.013 (C) 0.108 (D) 0.417 (E) 0.500

7. An auto insurance policy will pay for damage to both the policyholder's car and the other driver's car in the event that the policyholder is responsible for an accident. The size of the payment for damage to the policyholder's car, X, has a marginal density function of 1 for $0 < x < 1$. Given $X = x$, the size of the payment for damage to the other driver's car, Y, has conditional density of 1 for $x < y < x + 1$. If the policyholder is responsible for an accident, what is the probability that the payment for damage to the other driver's car will be greater than 0.500?

   (A) 3/8 (B) 1/2 (C) 3/4 (D) 7/8 (E) 15/16

8. A device that continuously measures and records seismic activity is placed in a remote region. The time, T, to failure of this device is exponentially distributed with mean 3 years. Since the device will not be monitored during its first two years of service, the time to discovery of its failure is $X = \max(T, 2)$. Determine $E(X)$.

   (A) $2 + \frac{1}{3}e^{-6}$

   (B) $2 - 2e^{-2/3} + 5e^{-4/3}$

   (C) 3

   (D) $2 + 3e^{-2/3}$

   (E) 5

9. The time, T, that a manufacturing system is out of operation has cumulative distribution function

$$F(t) = \begin{cases} 1 - \left(\frac{2}{t}\right)^2 & \text{for } t > 2, \\ 0 & \text{otherwise.} \end{cases}$$

The resulting cost to the company is $Y = T^2$. Determine the density function of Y, for $y > 4$.

(A) $4y^{-2}$ (B) $8y^{-3/2}$ (C) $8y^{-3}$ (D) $16y^{-1}$ (E) $1024y^{-5}$

10. Let X and Y be the number of hours that a randomly selected person watches movies and sporting events, respectively, during a three-month period. The following information is known about X and Y:

$$E(X) = 50, \quad E(Y) = 20, \quad \mathrm{Var}(X) = 50, \quad \mathrm{Var}(Y) = 30, \quad \mathrm{Cov}(X, Y) = 10.$$

One hundred people are randomly selected and observed for these three months. Let T be the total number of hours that these one hundred people watch movies or sporting events during this three-month period. Approximate the value of $P(T < 7100)$.

(A) 0.62 (B) 0.84 (C) 0.87 (D) 0.92 (E) 0.97

11. Once a fire is reported to a fire insurance company, the company makes an initial estimate, X, of the amount it will pay to the claimant for the fire loss. When the claim is finally settled, the company pays an amount, Y, to the claimant. The company has determined that X and Y have the joint density function

$$f(x,y) = \frac{2}{x^2(x-1)} y^{-(2x-1)/(x-1)}, \quad x > 1, y > 1.$$

Given that the initial claim estimated by the company is 2, determine the probability that the final settlement amount is between 1 and 3.

(A) 1/9 (B) 2/9 (C) 1/3 (D) 2/3 (E) 8/9

12. A company is reviewing tornado damage claims under a farm insurance policy. Let X be the portion of a claim representing damage to the house and let Y be the portion of the same claim representing damage to the rest of the property. The joint density function of X and Y is

$$f(x,y) = \begin{cases} 6(1-(x+y)) & \text{for } x>0, y>0, x+y<1, \\ 0 & \text{otherwise.} \end{cases}$$

Determine the probability that the portion of a claim representing damage to the house is less than 0.2.

(A) 0.360 (B) 0.480 (C) 0.488 (D) 0.512 (E) 0.520

13. The distribution of loss due to fire damage to a warehouse is:

Amount of loss	Probability
0	0.900
500	0.060
1,000	0.030
10,000	0.008
50,000	0.001
100,000	0.001

Given that a loss is greater than zero, calculate the expected amount of the loss.

(A) 290 (B) 322 (C) $1,704$ (D) $2,900$ (E) $32,222$

14. An auto insurance company has $10,000$ policyholders. Each policyholder is classified as

    (i) young or old;

    (ii) male or female; and

    (iii) married or single.

    Of these policyholders, 3000 are young, 4600 are male, and 7000 are married. The policyholders can also be classified as 1320 young males, 3010 married males, and 1400 young married persons. Finally, 600 of the policyholders are young married males. How many of the company's policyholders are young, female, and single?

    (A) 280 (B) 423 (C) 486 (D) 880 (E) 896

15. Two insurers provide bids on an insurance policy to a large company. The bids must be between 2000 and 2200. The company decides to accept the lower bid if the two bids differ by 20 or more. Otherwise, the company will consider the two bids further. Assume that the two bids are independent and are both uniformly distributed on the interval from 2000 to 2200. Determine the probability that the company considers the two bids further.

    (A) 0.10 (B) 0.19 (C) 0.20 (D) 0.41 (E) 0.60

16. The lifetime of a printer costing 200 is exponentially distributed with mean 2 years. The manufacturer agrees to pay a full refund to a buyer if the printer fails during the first year following its purchase, and a one-half refund if it fails during the second year. If the manufacturer sells 100 printers, how much should it expect to pay in refunds?

    (A) $6,321$ (B) $7,358$ (C) $7,869$ (D) $10,256$ (E) $12,642$

17. The monthly profit of Company I can be modeled by a continuous random variable with density function f. Company II has a monthly profit that is twice that of Company I. Determine the probability density function of the monthly profit of Company II.

    (A) $\frac{1}{2}f\left(\frac{x}{2}\right)$ (B) $f\left(\frac{x}{2}\right)$ (C) $2f\left(\frac{x}{2}\right)$ (D) $2f(x)$ (E) $2f(2x)$

18. An insurance policy is written to cover a loss X where X has density function

$$f(x) = \begin{cases} \frac{3}{8}x^2 & \text{for } 0 \le x \le 2, \\ 0 & \text{otherwise.} \end{cases}$$

The time (in hours) to process a claim of size x, where $0 \le x \le 2$, is uniformly distributed on the interval from x to $2x$. Calculate the probability that a randomly chosen claim on this policy is processed in three hours or more.

(A) 0.17 (B) 0.25 (C) 0.32 (D) 0.58 (E) 0.83

19. In an analysis of healthcare data, ages have been rounded to the nearest multiple of 5 years. The difference between the true age and the rounded age is assumed to be uniformly distributed on the interval from -2.5 years to 2.5 years. The healthcare data are based on a random sample of 48 people. What is the approximate probability that the mean of the rounded ages is within 0.25 years of the mean of the true ages?

(A) 0.14 (B) 0.38 (C) 0.57 (D) 0.77 (E) 0.88

20. The time to failure of a component in an electronic device has an exponential distribution with a median of four hours. Calculate the probability that the component will work without failing for at least five hours.

    (A) 0.07 (B) 0.29 (C) 0.38 (D) 0.42 (E) 0.57