Math 370/408, Spring 2008
Prof. A.J. Hildebrand

Actuarial Exam Practice Problem Set 5

About this problem set: These are problems from Course 1/P actuarial exams that I have collected over the years, grouped by topic, and ordered by difficulty. All of the exams have appeared on the SOA website `http://www.soa.org/` at some point in the past, though most of them are no longer there. The exams are copy-righted by the SOA.

Note on the topics covered: This problem set covers problems on the normal distribution and the Central Limit Theorem (Chapter 5 of Hogg/Tanis).

Note on the ordering of the problems: The problems are loosely grouped by topic, and very roughly ordered by difficulty (though that ordering is very subjective). The last group of problems (those with triple digit numbers) are harder, either because they are conceptually more difficult, or simply because they require lengthy computations and may take two or three times as long to solve than an average problem.

Answers and solutions: I will post an answer key and hints/solutions on the course webpage, `www.math.uiuc.edu/~hildebr/370`.

1. [5-1]

   Claims filed under auto insurance policies follow a normal distribution with mean $19,400$ and standard deviation $5,000$. What is the probability that the average of 25 randomly selected claims exceeds $20,000$?

   (A) 0.01 (B) 0.15 (C) 0.27 (D) 0.33 (E) 0.45

2. [5-2]

   The total claim amount for a health insurance policy follows a distribution with density function

   $$f(x) = \frac{1}{1000}e^{-x/1000}, \quad \text{for } x > 0.$$

   The premium for the policy is set at 100 over the expected total claim amount. If 100 policies are sold, what is the approximate probability that the insurance company will have claims exceeding the premiums collected?

   (A) 0.001 (B) 0.159 (C) 0.333 (D) 0.407 (E) 0.460

3. [5-3]

   In an analysis of healthcare data, ages have been rounded to the nearest multiple of 5 years. The difference between the true age and the rounded age is assumed to be uniformly distributed on the interval from -2.5 years to 2.5 years. The healthcare data are based on a random sample of 48 people. What is the approximate probability that the mean of the rounded ages is within 0.25 years of the mean of the true ages?

   (A) 0.14 (B) 0.38 (C) 0.57 (D) 0.77 (E) 0.88

4. [5-4]

   An insurance company issues 1250 vision care insurance policies. The number of claims filed by a policyholder under a vision care insurance policy during one year is a Poisson random variable with mean 2. Assume the numbers of claims filed by distinct policyholders are independent of one another. What is the approximate probability that there is a total of between 2450 and 2600 claims during a one-year period?

   (A) 0.68 (B) 0.82 (C) 0.87 (D) 0.95 (E) 1.00

5. [5-5]

   A charity receives 2025 contributions. Contributions are assumed to be independent and identically distributed with mean 3125 and standard deviation 250. Calculate the approximate 90th percentile for the distribution of the total contributions received.

   (A) $6,328,000$ (B) $6,338,000$ (C) $6,343,000$ (D) $6,784,000$ (E) $6,977,000$

6. [5-6]

   A company manufactures a brand of light bulb with a lifetime in months that is normally distributed with mean 3 and variance 1. A consumer buys a number of these bulbs with the intention of replacing them successively as they burn out. The light bulbs have independent lifetimes. What is the smallest number of bulbs to be purchased so that the succession of light bulbs produces light for at least 40 months with probability at least 0.9772 ?

   (A) 14 (B) 16 (C) 20 (D) 40 (E) 55

7. [5-7]

Two instruments are used to measure the height, h, of a tower. The error made by the less accurate instrument is normally distributed with mean 0 and standard deviation $0.0056h$. The error made by the more accurate instrument is normally distributed with mean 0 and standard deviation $0.0044h$. Assuming the two measurements are independent random variables, what is the probability that their average value is within $0.005h$ of the height of the tower?

(A) 0.38 (B) 0.47 (C) 0.68 (D) 0.84 (E) 0.90

8. [5-51]

Let X and Y be the number of hours that a randomly selected person watches movies and sporting events, respectively, during a three-month period. The following information is known about X and Y:

$$E(X) = 50, \quad E(Y) = 20, \quad \mathrm{Var}(X) = 50, \quad \mathrm{Var}(Y) = 30, \quad \mathrm{Cov}(X, Y) = 10.$$

One hundred people are randomly selected and observed for these three months. Let T be the total number of hours that these one hundred people watch movies or sporting events during this three-month period. Approximate the value of $P(T < 7100)$.

(A) 0.62 (B) 0.84 (C) 0.87 (D) 0.92 (E) 0.97

9. [5-52]

A city has just added 100 new female recruits to its police force. The city will provide a pension to each new hire who remains with the force until retirement. In addition, if the new hire is married at the time of her retirement, a second pension will be provided for her husband. A consulting actuary makes the following assumptions:

(i) Each new recruit has a 0.4 probability of remaining with the police force until retirement.

(ii) Given that a new recruit reaches retirement with the police force, the probability that she is not married at the time of retirement is 0.25.

(iii) The number of pensions that the city will provide on behalf of each new hire is independent of the number of pensions it will provide on behalf of any other new hire.

Determine the probability that the city will provide at most 90 pensions to the 100 new hires and their husbands.

(A) 0.60 (B) 0.67 (C) 0.75 (D) 0.93 (E) 0.99

10. [5-102]

For Company A there is a 60% chance that no claim is made during the coming year. If one or more claims are made, the total claim amount is normally distributed with mean $10,000$ and standard deviation $2,000$. For Company B there is a 70% chance that no claim is made during the coming year. If one or more claims are made, the total claim amount is normally distributed with mean $9,000$ and standard deviation $2,000$. Assume that the total claim amounts of the two companies are independent. What is the probability that, in the coming year, Company B's total claim amount will exceed Company A's total claim amount?

(A) 0.180 (B) 0.185 (C) 0.217 (D) 0.223 (E) 0.240