Math 370/408, Spring 2008
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Actuarial Exam Practice Problem Set 3

About this problem set: These are problems from Course 1/P actuarial exams that I have collected over the years, grouped by topic, and ordered by difficulty. All of the exams have appeared on the SOA website `http://www.soa.org/` at some point in the past, though most of them are no longer there. The exams are copy-righted by the SOA.

Note on the topics covered: This problem set covers problems on continuous random variables (Chapter 3 of Hogg/Tanis).

Note on the ordering of the problems: The problems are loosely grouped by topic, and very roughly ordered by difficulty (though that ordering is very subjective). The last group of problems (those with triple digit numbers) are harder, either because they are conceptually more difficult, or simply because they require lengthy computations and may take two or three times as long to solve than an average problem.

Answers and solutions: I will post an answer key and hints/solutions on the course webpage, `www.math.uiuc.edu/~hildebr/370`.

1. [3-1]

An insurance company issued insurance policies to 32 independent risks. For each policy, the probability of a claim is 1/6. The benefit given that there is a claim has probability density function

$$f(y) = \begin{cases} 2(1-y), & 0 < y < 1, \\ 0, & \text{otherwise.} \end{cases}$$

Calculate the expected value of total benefits paid.

(A) $\frac{16}{9}$ (B) $\frac{8}{3}$ (C) $\frac{32}{9}$ (D) $\frac{16}{3}$ (E) $\frac{32}{3}$

2. [3-2]

Let X be a continuous random variable with density function

$$f(x) = \begin{cases} \frac{|x|}{10} & \text{for } -2 \le x \le 4, \\ 0 & \text{otherwise.} \end{cases}$$

Calculate the expected value of X.

(A) $\frac{1}{5}$ (B) $\frac{3}{5}$ (C) 1 (D) $\frac{28}{15}$ (E) $\frac{12}{5}$

3. [3-3]

A large company has determined that the function

$$f(x) = 3x^2, \quad 0 \le x \le 1,$$

serves as the payroll density function. That is, the distribution of payroll, $F(x)$, which is the proportion of total payroll earned by the lowest paid fraction x of employees, $0 \le x \le 1$, relates to $f(x)$ in the same way that probability distributions and probability densities relate. Gini's index, G, defined as

$$G = 2\int_0^1 |x - F(x)| dx$$

is a measure of how evenly payroll is distributed among all employees. Calculate G for this large company.

(A) 0.2 (B) 0.4 (C) 0.5 (D) 0.8 (E) 1.0

4. [3-4]

The loss amount, X, for a medical insurance policy has cumulative distribution function

$$F(x) = \begin{cases} 0, & x < 0, \\ \frac{1}{9}\left(2x^2 - \frac{x^3}{3}\right), & 0 \le x \le 3, \\ 1, & x > 3. \end{cases}$$

Calculate the mode of the distribution.

(A) $\frac{2}{3}$ (B) 1 (C) $\frac{3}{2}$ (D) 2 (E) 3

5. [3-5]

   An insurance company's monthly claims are modeled by a continuous, positive random variable X, whose probability density function is proportional to $(1+x)^{-4}$, where $0 < x < \infty$. Determine the company's expected monthly claims.

   (A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) 1 (E) 3

6. [3-6]

   An insurance policy reimburses dental expense, X, up to a maximum benefit of 250. The probability density function for X is

   $$f(x) = \begin{cases} ce^{-0.004x} & \text{for } x \geq 0, \\ 0 & \text{otherwise,} \end{cases}$$

   where c is a constant. Calculate the median benefit for this policy.

   (A) 161 (B) 165 (C) 173 (D) 182 (E) 250

7. [3-7]

   The time to failure of a component in an electronic device has an exponential distribution with a median of four hours. Calculate the probability that the component will work without failing for at least five hours.

   (A) 0.07 (B) 0.29 (C) 0.38 (D) 0.42 (E) 0.57

8. [3-8]

   The number of days that elapse between the beginning of a calendar year and the moment a high-risk driver is involved in an accident is exponentially distributed. An insurance company expects that 30% of high-risk drivers will be involved in an accident during the first 50 days of a calendar year. What portion of high-risk drivers are expected to be involved in an accident during the first 80 days of a calendar year?

   (A) 0.15 (B) 0.34 (C) 0.43 (D) 0.57 (E) 0.66

9. [3-9]

   The monthly profit of Company I can be modeled by a continuous random variable with density function f. Company II has a monthly profit that is twice that of Company I. Determine the probability density function of the monthly profit of Company II.

   (A) $\frac{1}{2}f\left(\frac{x}{2}\right)$ (B) $f\left(\frac{x}{2}\right)$ (C) $2f\left(\frac{x}{2}\right)$ (D) $2f(x)$ (E) $2f(2x)$

10. [3-10]

    An insurance company insures a large number of homes. The insured value, X, of a randomly selected home is assumed to follow a distribution with density function

    $$f(x) = \begin{cases} 3x^{-4} & \text{for } x > 1, \\ 0 & \text{otherwise.} \end{cases}$$

    Given that a randomly selected home is insured for at least 1.5, what is the probability that it is insured for less than 2?

    (A) 0.578 (B) 0.684 (C) 0.704 (D) 0.829 (E) 0.875

11. [3-11]

An insurance policy reimburses a loss up to a benefit limit of 10. The policyholder's loss, Y, follows a distribution with density function

$$f(y) = \begin{cases} 2y^{-3} & \text{for } y > 1, \\ 0 & \text{otherwise.} \end{cases}$$

What is the expected value of the benefit paid under the insurance policy?

(A) 1.0 (B) 1.3 (C) 1.8 (D) 1.9 (E) 2.0

12. [3-12]

The lifetime of a machine part has a continuous distribution on the interval $(0, 40)$ with probability density function f, where $f(x)$ is proportional to $(10 + x)^{-2}$. Calculate the probability that the lifetime of the machine part is less than 6.

(A) 0.04 (B) 0.15 (C) 0.47 (D) 0.53 (E) 0.94

13. [3-13]

The time, T, that a manufacturing system is out of operation has cumulative distribution function

$$F(t) = \begin{cases} 1 - \left(\frac{2}{t}\right)^2 & \text{for } t > 2, \\ 0 & \text{otherwise.} \end{cases}$$

The resulting cost to the company is $Y = T^2$. Determine the density function of Y, for $y > 4$.

(A) $4y^{-2}$ (B) $8y^{-3/2}$ (C) $8y^{-3}$ (D) $16y^{-1}$ (E) $1024y^{-5}$

14. [3-14]

Under a group insurance policy, an insurer agrees to pay 100% of the medical bills incurred during the year by employees of a small company, up to a maximum total of one million dollars. The total amount of bills incurred, X, has probability density function

$$f(x) = \begin{cases} \frac{x(4-x)}{9} & \text{for } 0 < x < 3, \\ 0 & \text{otherwise,} \end{cases}$$

where x is measured in millions. Calculate the total amount, in millions of dollars, the insurer would expect to pay under this policy.

(A) 0.120 (B) 0.301 (C) 0.935 (D) 2.338 (E) 3.495

15. [3-15]

The lifetime of a printer costing 200 is exponentially distributed with mean 2 years. The manufacturer agrees to pay a full refund to a buyer if the printer fails during the first year following its purchase, and a one-half refund if it fails during the second year. If the manufacturer sells 100 printers, how much should it expect to pay in refunds?

(A) $6,321$ (B) $7,358$ (C) $7,869$ (D) $10,256$ (E) $12,642$

16. [3-16]

An insurance policy is written to cover a loss, X, where X has a uniform distribution on $[0, 1000]$. At what level must a deductible be set in order for the expected payment to be 25% of what it would be with no deductible?

(A) 250 (B) 375 (C) 500 (D) 625 (E) 750

17. [3-17]

An insurance company sells an auto insurance policy that covers losses incurred by a policyholder, subject to a deductible of 100. Losses incurred follow an exponential distribution with mean 300. What is the 95th percentile of actual losses that exceed the deductible?

(A) 600 (B) 700 (C) 800 (D) 900 (E) 1000

18. [3-18]

A piece of equipment is being insured against early failure. The time from purchase until failure of the equipment is exponentially distributed with mean 10 years. The insurance will pay an amount x if the equipment fails during the first year, and it will pay $0.5x$ if failure occurs during the second or third year. If failure occurs after the first three years, no payment will be made. At what level must x be set if the expected payment made under this insurance is to be 1000?

(A) 3858 (B) 4449 (C) 5382 (D) 5644 (E) 7235

19. [3-19]

A manufacturer's annual losses follow a distribution with density function

$$f(x) = \begin{cases} \frac{2.5(0.6)^{2.5}}{x^{3.5}} & \text{for } x > 0.6, \\ 0 & \text{otherwise.} \end{cases}$$

To cover its losses, the manufacturer purchases an insurance policy with an annual deductible of 2. What is the mean of the manufacturer's annual losses not paid by the insurance policy?

(A) 0.84 (B) 0.88 (C) 0.93 (D) 0.95 (E) 1.00

20. [3-20]

A device that continuously measures and records seismic activity is placed in a remote region. The time, T, to failure of this device is exponentially distributed with mean 3 years. Since the device will not be monitored during its first two years of service, the time to discovery of its failure is $X = \max(T, 2)$. Determine $E(X)$.

(A) $2 + \frac{1}{3}e^{-6}$

(B) $2 - 2e^{-2/3} + 5e^{-4/3}$

(C) 3

(D) $2 + 3e^{-2/3}$

(E) 5

21. [3-21]

An insurance policy pays for a random loss X subject to a deductible of C, where $0 < C < 1$. The loss amount is modeled as a continuous random variable with density function

$$f(x) = \begin{cases} 2x & \text{for } 0 < x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

Given a random loss X, the probability that the insurance payment is less than 0.5 is equal to 0.64. Calculate C.

(A) 0.1 (B) 0.3 (C) 0.4 (D) 0.6 (E) 0.8

22. [3-22]

An actuary is reviewing a study she performed on the size of the claims made ten years ago under homeowners insurance policies. In her study, she concluded that the size of claims followed an exponential distribution and that the probability that a claim would be less than $1,000 was 0.250. The actuary feels that the conclusions she reached in her study are still valid today with one exception: every claim made today would be twice the size of a similar claim made ten years ago as a result of inflation. Calculate the probability that the size of a claim made today is less than $1,000.

23. [3-23]

A group insurance policy covers the medical claims of the employees of a small company. The value, V, of the claims made in one year is described by $V = 100,000Y$, where Y is a random variable with density function

$$f(y) = \begin{cases} k(1-y)^4 & \text{for } 0 < y < 1, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant. What is the conditional probability that V exceeds $40,000$, given that V exceeds $10,000$?

(A) 0.08 (B) 0.13 (C) 0.17 (D) 0.20 (E) 0.51

24. [3-24]

The value, ν, of an appliance is given by $\nu(t) = e^{7-0.2t}$, where t denotes the number of years since purchase. If appliance fails within seven years of purchase, a warranty pays the owner the value of the appliance at the time of failure. After seven years, the warranty pays nothing. The time until failure of the appliance has an exponential distribution with mean 10. Calculate the expected payment from the warranty.

(A) 98.70 (B) 109.66 (C) 270.43 (D) 320.78 (E) 352.16

25. [3-51]

The loss due to a fire in a commercial building is modeled by a random variable X with density function

$$f(x) = \begin{cases} 0.005(20-x) & \text{for } 0 < x < 20, \\ 0 & \text{otherwise.} \end{cases}$$

Given that a fire loss exceeds 8, what is the probability that it exceeds 16?

(A) 1/25 (B) 1/9 (C) 1/8 (D) 1/3 (E) 3/7

26. [3-52]

An insurer's annual weather-related loss, X, is a random variable with density function

$$f(x) = \begin{cases} \dfrac{2.5(200)^{2.5}}{x^{3.5}} & \text{for } x > 200, \\ 0 & \text{otherwise.} \end{cases}$$

Calculate the difference between the 30th and 70th percentiles of X.

(A) 35 (B) 93 (C) 124 (D) 231 (E) 298

27. [3-53]

The warranty on a machine specifies that it will be replaced at failure or age 4, whichever occurs first. The machine's age at failure, X, has density function

$$f(x) = \begin{cases} 1/5 & \text{for } 0 < x < 5, \\ 0 & \text{otherwise.} \end{cases}$$

Let Y be the age of the machine at the time of replacement. Determine the variance of Y.

(A) 1.3 (B) 1.4 (C) 1.7 (D) 2.1 (E) 7.5

28. [3-54]

The owner of an automobile insures it against damage by purchasing an insurance policy with a deductible of 250. In the event that the automobile is damaged, repair costs can be modeled by a uniform random variable on the interval $(0, 1500)$. Determine the standard deviation of the insurance payment in the event that the automobile is damaged.

(A) 361 (B) 403 (C) 433 (D) 464 (E) 521

29. [3-55]

An actuary models the lifetime of a device using the random variable $Y = 10X^{0.8}$, where X is an exponential random variable with mean 1 year. Determine the probability density function $f(y)$, for $y > 0$, of the random variable Y.

(A) $10y^{0.8}e^{-8y^{-0.2}}$

(B) $8y^{-0.2}e^{-10y^{0.8}}$

(C) $8y^{-0.2}e^{-0.1y^{1.25}}$

(D) $(0.1y)^{1.25}e^{-0.125(0.1y)^{0.25}}$

(E) $0.125(0.1y)^{0.25}e^{-(0.1y)^{1.25}}$

30. [3-56]

A tour operator has a bus that can accommodate 20 tourists. The operator knows that tourists may not show up, so he sells 21 tickets. The probability that an individual tourist will not show up is 0.02, independent of all other tourists. Each ticket costs 50, and is non-refundable if a tourist fails to show up. If a tourist shows up and a seat is not available, the tour operator has to pay 100 (ticket cost + 50 penalty) to the tourist. What is the expected revenue of the tour operator?

(A) 935 (B) 950 (C) 967 (D) 976 (E) 985

31. [3-57]

An investment account earns an annual interest rate R that follows a uniform distribution on the interval $(0.04, 0.08)$. The value of a $10,000$ initial investment in this account after one year is given by $V = 10,000e^R$. Determine the cumulative distribution function, $F(v)$, of V for values v that satisfy $0 < F(v) < 1$.

(A) $\dfrac{10,000e^{v/10,000} - 10,408}{425}$

(B) $25e^{v/10,000} - 0.04$

(C) $\dfrac{v - 10,408}{10,833 - 10,408}$

(D) $\dfrac{25}{v}$

(E) $25\left(\ln\left(\dfrac{v}{10,000}\right) - 0.04\right)$

32. [3-61]

You are given the following information about N, the annual number of claims for a randomly selected insured:

$$P(N=0) = \frac{1}{2}, \quad P(N=1) = \frac{1}{3}, \quad P(N>1) = \frac{1}{6}.$$

Let S denote the total annual claim amount for an insured. When $N = 1$, S is exponentially distributed with mean 5. When $N > 1$, S is exponentially distributed with mean 8. Determine $P(4 < S < 8)$.

(A) 0.04 (B) 0.08 (C) 0.12 (D) 0.24 (E) 0.25

33. [3-102]

Let T denote the time in minutes for a customer service representative to respond to 10 telephone inquiries. T is uniformly distributed on the interval with endpoints 8 minutes and 12 minutes. Let R denote the average rate, in customers per minute, at which the representative responds to inquiries. Which of the following is the density function of the random variable R on the interval $(10/12 \le r \le 10/8)$?

(A) $\frac{12}{5}$

(B) $3 - \frac{5}{2r}$

(C) $3r - \frac{5\ln r}{2}$

(D) $\frac{10}{r^2}$

(E) $\frac{5}{2r^2}$

34. [3-103]

An auto insurance company insures an automobile worth $15,000$ for one year under a policy with a $1,000$ deductible. During the policy year there is a 0.04 chance of partial damage to the car and a 0.02 chance of a total loss of the car. If there is partial damage to the car, the amount X of damage (in thousands) follows a distribution with density function

$$f(x) = \begin{cases} 0.5003e^{-x/2} & \text{for } 0 < x < 15, \\ 0 & \text{otherwise.} \end{cases}$$

What is the expected claim payment?

(A) 320 (B) 328 (C) 352 (D) 380 (E) 540