Math 370/408, Spring 2008
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Actuarial Exam Practice Problem Set 2

About this problem set: These are problems from Course 1/P actuarial exams that I have collected over the years, grouped by topic, and ordered by difficulty. All of the exams have appeared on the SOA website `http://www.soa.org/` at some point in the past, though most of them are no longer there. The exams are copy-righted by the SOA.

Note on the topics covered: This problem set covers discrete random variables (Chapter 2 of Hogg/Tanis).

Note on the ordering of the problems: The problems are loosely grouped by topic, and very roughly ordered by difficulty (though that ordering is very subjective). The last group of problems (those with triple digit numbers) are harder, either because they are conceptually more difficult, or simply because they require lengthy computations and may take two or three times as long to solve than an average problem.

Answers and solutions: I will post an answer key and hints/solutions on the course webpage, `www.math.uiuc.edu/~hildebr/370`.

1. [2-1]

   An insurance policy pays an individual 100 per day for up to 3 days of hospitalization and 25 per day for each day of hospitalization thereafter. The number of days of hospitalization, X, is a discrete random variable with probability function

   $$P(X = k) = \begin{cases} \frac{6-k}{15} & \text{for } k = 1, 2, 3, 4, 5, \\ 0 & \text{otherwise.} \end{cases}$$

   Calculate the expected payment for hospitalization under this policy.

   (A) 85 (B) 163 (C) 168 (D) 213 (E) 255

2. [2-2]

   An actuary determines that the claim size for a certain class of accidents is a random variable, X, with moment generating function

   $$M_X(t) = \frac{1}{(1 - 2500t)^4}.$$

   Determine the standard deviation of the claim size for this class of accidents.

   (A) $1,340$ (B) $5,000$ (C) $8,660$ (D) $10,000$ (E) $11,180$

3. [2-3]

   An actuary has discovered that policyholders are three times as likely to file two claims as to file four claims. If the number of claims filed has a Poisson distribution, what is the variance of the number of claims filed?

   (A) $1/\sqrt{3}$ (B) 1 (C) $\sqrt{2}$ (D) 2 (E) 4

4. [2-4]

   In modeling the number of claims filed by an individual under an automobile policy during a three-year period, an actuary makes the simplifying assumption that for all integers $n \geq 0$, $p_{n+1} = (1/5)p_n$, where p_n represents the probability that the policyholder files n claims during the period. Under this assumption, what is the probability that a policyholder files more than one claim during the period?

   (A) 0.04 (B) 0.16 (C) 0.20 (D) 0.80 (E) 0.96

5. [2-5]

   As part of the underwriting process for insurance, each prospective policyholder is tested for high blood pressure. Let X represent the number of tests completed when the first person with high blood pressure is found. The expected value of X is 12.5. Calculate the probability that the sixth person tested is the first one with high blood pressure.

   (A) 0.000 (B) 0.053 (C) 0.080 (D) 0.316 (E) 0.394

6. [2-6]

   Let X be a random variable with moment generating function

   $$M(t) = \left(\frac{2 + e^t}{3}\right)^9, \quad -\infty < t < +\infty.$$

Calculate the variance of X.

(A) 2 (B) 3 (C) 8 (D) 9 (E) 11

7. [2-7]

A small commuter plane has 30 seats. The probability that any particular passenger will not show up for a flight is 0.10, independent of other passengers. The airline sells 32 tickets for the flight. Calculate the probability that more passengers show up for the flight than there are seats available.

(A) 0.0042 (B) 0.0343 (C) 0.0382 (D) 0.1221 (E) 0.1564

8. [2-8]

The distribution of loss due to fire damage to a warehouse is:

Amount of loss	Probability
0	0.900
500	0.060
1,000	0.030
10,000	0.008
50,000	0.001
100,000	0.001

Given that a loss is greater than zero, calculate the expected amount of the loss.

(A) 290 (B) 322 (C) $1,704$ (D) $2,900$ (E) $32,222$

9. [2-9]

Let X_1, X_2, X_3 be a random sample from a discrete distribution with probability function

$$p(x) = \begin{cases} \frac{1}{3} & \text{for } x = 0, \\ \frac{2}{3} & \text{for } x = 1, \\ 0 & \text{otherwise.} \end{cases}$$

Determine the moment generating function, $M(t)$, of $Y = X_1X_2X_3$.

(A) $\frac{19}{27} + \frac{8}{27}e^t$
(B) $1 + 2e^t$
(C) $\left(\frac{1}{3} + \frac{2}{3}e^t\right)^3$
(D) $\frac{1}{27} + \frac{8}{27}e^{3t}$
(E) $\frac{1}{3} + \frac{2}{3}e^{3t}$

10. [2-51]

The number of injury claims per month is modeled by a random variable N with

$$P(N = n) = \frac{1}{(n+1)(n+2)}, \quad \text{where } n \geq 0.$$

Determine the probability of at least one claim during a particular month, given that there have been at most four claims during that month.

(A) 1/3 (B) 2/5 (C) 1/2 (D) 3/5 (E) 5/6

11. [2-52]

An insurance company determines that N, the number of claims received in a week, is a random variable with $P(N = n) = 2^{-n-1}$, where $n \geq 0$. The company also determines that the number of claims received in a given week is independent of the number of claims received in any other week. Determine the probability that exactly seven claims will be received during a given two-week period.

(A) $\frac{1}{256}$ (B) $\frac{1}{128}$ (C) $\frac{7}{512}$ (D) $\frac{1}{64}$ (E) $\frac{1}{32}$

12. [2-53]

A company prices its hurricane insurance using the following assumptions:

(i) In any calendar year, there can be at most one hurricane.
(ii) In any calendar year, the probability of a hurricane is 0.05.
(iii) The number of hurricanes in any calendar year is independent of the number of hurricanes in any other calendar year.

Using the company's assumptions, calculate the probability that there are fewer than 3 hurricanes in a 20-year period.

(A) 0.06 (B) 0.19 (C) 0.38 (D) 0.62 (E) 0.92

13. [2-54]

An insurance policy on an electrical device pays a benefit of 4000 if the device fails during the first year. The amount of the benefit decreases by 1000 each successive year until it reaches 0. If the device has not failed by the beginning of any given year, the probability of failure during that year is 0.4. What is the expected benefit under this policy?

(A) 2234 (B) 2400 (C) 2500 (D) 2667 (E) 2694

14. [2-55]

An insurance company sells a one-year automobile policy with a deductible of 2. The probability that the insured will incur a loss is 0.05. If there is a loss, the probability of a loss of amount N is K/N, for $N = 1, \ldots, 5$ and K a constant. These are the only possible loss amounts and no more than one loss can occur. Determine the net premium for this policy.

(A) 0.031 (B) 0.066 (C) 0.072 (D) 0.110 (E) 0.150

15. [2-56]

A probability distribution of the claim sizes for an auto insurance policy is given in the table below:

Claim size	Probability
20	0.15
30	0.10
40	0.05
50	0.20
60	0.10
70	0.10
80	0.30

What percentage of the claims are within one standard deviation of the mean claim size?

(A) 45% (B) 55% (C) 68% (D) 85% (E) 100%

16. [2-60]

Two life insurance policies, each with a death benefit of $10,000$ and a one-time premium of 500, are sold to a couple, one for each person. The policies will expire at the end of the tenth year. The probability that only the wife will survive at least ten years is 0.025, the probability that only the husband will survive at least ten years is 0.01, and the probability that both of them will survive at least ten years is 0.96. What is the expected excess of premiums over claims, given that the husband survives at least ten years?

(A) 350 (B) 385 (C) 397 (D) 870 (E) 897

17. [2-101]

A company establishes a fund of 120 from which it wants to pay an amount, C, to any of its 20 employees who achieve a high performance level during the coming year. Each employee has a 2% chance of achieving a high performance level during the coming year, independent of any other employee. Determine the maximum value of C for which the probability is less than 1% that the fund will be inadequate to cover all payments for high performance.

(A) 24 (B) 30 (C) 40 (D) 60 (E) 120

18. [2-102]

A company takes out an insurance policy to cover accidents that occur at its manufacturing plant. The probability that one or more accidents will occur during any given month is 3/5. The number of accidents that occur in any given month is independent of the number of accidents that occur in all other months. Calculate the probability that there will be at least four months in which no accidents occur before the fourth month in which at least one accident occurs.

(A) 0.01 (B) 0.12 (C) 0.23 (D) 0.29 (E) 0.41

19. [2-103]

A company buys a policy to insure its revenue in the event of major snowstorms that shut down business. The policy pays nothing for the first such snowstorm of the year and $10,000$ for each one therefore, until the end of the year. The number of major snowstorms per year that shut down business is assumed to have Poisson distribution with mean 1.5. What is the expected amount paid to the company under this policy during a one-year period?

(A) $2,769$ (B) $5,000$ (C) $7,231$ (D) $8,347$ (E) $10,578$

20. [2-104]

A study is being conducted in which the health of two independent groups of ten policyholders is being monitored over a one-year period of time. Individual participants in the study drop out before the end of the study with probability 0.2 (independently of the other participants). What is the probability that at least 9 participants complete the study in one of the two groups, but not in both groups?

(A) 0.096 (B) 0.192 (C) 0.235 (D) 0.376 (E) 0.469