Math 370/408, Spring 2008
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Actuarial Exam Practice Problem Set 1

About this problem set: These are problems from Course 1/P actuarial exams that I have collected over the years, grouped by topic, and ordered by difficulty. All of the exams have appeared on the SOA website `http://www.soa.org/` at some point in the past, though most of them are no longer there. The exams are copy-righted by the SOA.

Note on the topics covered: The topics are those covered in Chapter 1 of Hogg/Tanis, with the exception of the combinatorial probability problems from Section 1.3: set-theoretic properties and rules, general rules of probability, independence and conditional probability, and Bayes' Rule.

Note on the ordering of the problems: The problems are loosely grouped by topic, and very roughly ordered by difficulty (though that ordering is very subjective). The last group of problems (those with triple digit numbers) are harder, either because they are conceptually more difficult, or simply because they require lengthy computations and may take two or three times as long to solve than an average problem.

Answers and solutions: I will post an answer key and hints/solutions on the course webpage, `www.math.uiuc.edu/~hildebr/370`.

1. [1-1]

   A marketing survey indicates that 60% of the population owns an automobile, 30% owns a house, and 20% owns both an automobile and a house. Calculate the probability that a person chosen at random owns an automobile or a house, but not both.

   (A) 0.4 (B) 0.5 (C) 0.6 (D) 07 (E) 09

2. [1-2]

   The probability that a visit to a primary care physician's (PCP) office results in neither lab work nor referral to a specialist is 35%. Of those coming to a PCP's office, 30% are referred to specialists and 40% require lab work. Determine the probability that a visit to a PCP's office results in both lab work and referral to a specialist.

   (A) 0.05 (B) 0.12 (C) 0.18 (D) 0.25 (E) 0.35

3. [1-3]

   You are given $P(A \cup B) = 0.7$ and $P(A \cup B') = 0.9$. Determine $P(A)$.

   (A) 0.2 (B) 0.3 (C) 0.4 (D) 0.6 (E) 0.8

4. [1-4]

   Workplace accidents are categorized into three groups: minor, moderate, severe. The probability that a given accident is minor is 0.5, that it is moderate is 0.4, and that it is severe is 0.1. Two accidents occur independently in one month. Calculate the probability that neither accident is severe and at most one is moderate.

   (A) 0.25 (B) 0.40 (C) 0.45 (D) 0.56 (E) 0.65

5. [1-5]

   An actuary studying the insurance preferences of automobile owners makes the following conclusions:

   (i) An automobile owner is twice as likely to purchase collision coverage as disability coverage.
   (ii) The event that an automobile owner purchases collision coverage is independent of the event that he or she purchases disability coverage.
   (iii) The probability that an automobile owner purchases both collision and disability coverages is 0.15.

   What is the probability that an automobile owner purchases neither collision nor disability coverage?

   (A) 0.18 (B) 0.33 (C) 0.48 (D) 0.67 (E) 0.82

6. [1-6]

   An insurance company estimates that 40% of policyholders who have only an auto policy will renew next year and 60% of policyholders who have only a homeowners policy will renew next year. The company estimates that 80% of policyholders who have both an auto and a homeowners policy will renew at least one of those policies next year. Company records show that 65% of policyholders have an auto policy, 50% of policyholders have a homeowners policy, and 15% of policyholders have both an auto and a homeowners policy. Using the company's estimates, calculate the percentage of policyholders that will renew at least one policy next year.

(A) 20 (B) 29 (C) 41 (D) 53 (E) 70

7. [1-7]

A survey of a group's viewing habits over the last year revealed the following information:

(i) 28% watched gymnastics.
(ii) 29% watched baseball.
(iii) 19% watched soccer.
(iv) 14% watched gymnastics and baseball.
(v) 12% watched baseball and soccer.
(vi) 10% watched gymnastics and soccer.
(vii) 8% watched all three sports.

Calculate the percentage of the group that watched none of the three sports during the last year.

(A) 24 (B) 36 (C) 41 (D) 52 (E) 60

8. [1-8]

An insurance company pays hospital claims. The number of claims that include emergency room or operating room charges is 85% of the total number of claims. The number of claims that do not include emergency room charges is 25% of the total number of claims. The occurrence of emergency room charges is independent of the occurrence of operating room charges on hospital claims. Calculate the probability that a claim submitted to the insurance company includes operating room charges.

(A) 0.10 (B) 0.20 (C) 0.25 (D) 0.40 (E) 0.80

9. [1-9]

A blood test indicates the presence of a particular disease 95% of the time when the disease is actually present. The same test indicates the presence of the disease 0.5% of the time when the disease is not present. One percent of the population actually has the disease. Calculate the probability that a person has the disease given that the test indicates the presence of the disease.

(A) 0.324 (B) 0.657 (C) 0.945 (D) 0.950 (E) 0.995

10. [1-10]

Ten percent of a company's life insurance policyholders are smokers. The rest are nonsmokers. For each nonsmoker, the probability of dying during the year is 0.01. For each smoker, the probability of dying during the year is 0.05. Given that a policyholder has died, what is the probability that the policyholder was a smoker?

(A) 0.05 (B) 0.20 (C) 0.36 (D) 0.56 (E) 0.90

11. [1-11]

An auto insurance company insures drivers of all ages. An actuary compiled the following statistics on the company's insured drivers:

Age of driver	Probability of accident	Portion of company's insured drivers
16–20	0.06	0.08
21–30	0.03	0.15
31–65	0.02	0.49
66–99	0.04	0.28

A randomly selected driver that the company insures has an accident. Calculate the probability that the driver was age 16-20.

(A) 0.13 (B) 0.16 (C) 0.19 (D) 0.23 (E) 0.40

12. [1-12]

An actuary studied the likelihood that different types of drivers would be involved in at least one collision during any one-year period. The results of the study are presented below.

Type of driver	Percentage of all drivers	Probability of at least one collision
Teen	8%	0.15
Yound adult	16%	0.08
Midlife	45%	0.04
Senior	31%	0.05

Given that a driver has been involved in at least one collision in the past year, what is the probability that the driver is a young adult driver?

(A) 0.06 (B) 0.16 (C) 0.19 (D) 0.22 (E) 0.25

13. [1-13]

A health study tracked a group of persons for five years. At the beginning of the study, 20% were classified as heavy smokers, 30% as light smokers, and 50% as nonsmokers. Results of the study showed that light smokers were twice as likely as nonsmokers to die during the five-year study, but only half as likely as heavy smokers. A randomly selected participant from the study died over the five-year period. Calculate the probability that the participant was a heavy smoker.

(A) 0.20 (B) 0.25 (C) 0.35 (D) 0.42 (E) 0.57

14. [1-14]

The probability that a randomly chosen male has a circulation problem is 0.25. Males who have a circulation problem are twice as likely to be smokers as those who do not have a circulation problem. What is the conditional probability that a male has a circulation problem, given that he is a smoker?

(A) 1/4 (B) 1/3 (C) 2/5 (D) 1/2 (E) 2/3

15. [1-19]

A doctor is studying the relationship between blood pressure and heartbeat abnormalities in her patients. She tests a random sample of her patients and notes their blood pressures (high, low, or normal) and their heartbeats (regular or irregular). She finds that:

(i) 14% have high blood pressure.
(ii) 22% have low blood pressure.
(iii) 15% have an irregular heartbeat.
(iv) Of those with an irregular heartbeat, one-third have high blood pressure.
(v) Of those with normal blood pressure, one-eighth have an irregular heartbeat.

What portion of the patients selected have a regular heartbeat and low blood pressure?

(A) 2% (B) 5% (C) 8% (D) 9% (E) 20%

16. [1-51]

An urn contains 10 balls, 4 red and 6 blue. A second urn contains 16 red balls and an unknown number of blue balls. A single ball is drawn from each urn. The probability that both balls are of the same color is 0.44. Calculate the number of blue balls in the second urn.

(A) 4 (B) 20 (C) 24 (D) 44 (E) 64

17. [1-52]

A public health researcher examines the medical records of a group of 937 men who died in 1999 and discovers that 210 of the men died from causes related to heart disease. Moreover, 312 of the 937 men had at least one parent who suffered from heart disease, and, of these 312 men, 102 died from causes related to heart disease. Determine the probability that a man randomly selected from this group died of causes related to heart disease, given that neither of his parents suffered from heart disease.

(A) 0.115 (B) 0.173 (C) 0.224 (D) 0.327 (E) 0.514

18. [1-53]

Among a large group of patients recovering from shoulder injuries, it is found that 22% visit both a physical therapist and a chiropractor, whereas 12% visit neither of these. The probability that a patient visits a chiropractor exceeds by 0.14 the probability that a patient visits a physical therapist. Determine the probability that a randomly chosen member of this group visits a physical therapist.

(A) 0.26 (B) 0.38 (C) 0.40 (D) 0.48 (E) 0.62

19. [1-54]

    An insurer offers a health plan to the employees of a large company. As part of this plan, the individual employees may choose exactly two of the supplementary coverages A, B, and C, or they may choose no supplementary coverage. The proportions of the company's employees that choose coverages A, B, and C are 1/4, 1/3, and 5/12, respectively. Determine the probability that a randomly chosen employee will choose no supplementary coverage.

    (A) 0 (B) $\frac{47}{144}$ (C) $\frac{1}{2}$ (D) $\frac{97}{144}$ (E) $\frac{7}{9}$

20. [1-55]

    An insurance company issues life insurance policies in three separate categories: standard, preferred, and ultra-preferred. Of the company's policyholders, 50% are standard, 40% are preferred, and 10% are ultra-preferred. Each standard policyholder has probability 0.010 of dying in the next year, each preferred policyholder has probability 0.005 of dying in the next year, and each ultra-preferred policyholder has probability 0.001 of dying in the next year. A policyholder dies in the next year. What is the probability that the deceased policyholder was ultra-preferred?

    (A) 0.0001 (B) 0.0010 (C) 0.0071 (D) 0.0141 (E) 0.2817

21. [1-56]

    Upon arrival at a hospital's emergency room, patients are categorized according to their condition as critical, serious, or stable. In the past year:

    (i) 10% of the emergency room patients were critical;
    (ii) 30% of the emergency room patients were serious;
    (iii) the rest of the emergency room patients were stable;
    (iv) 40% of the critical patients died;
    (vi) 10% of the serious patients died; and
    (vii) 1% of the stable patients died.

    Given that a patient survived, what is the probability that the patient was categorized as serious upon arrival?

    (A) 0.06 (B) 0.29 (C) 0.30 (D) 0.39 (E) 0.64

22. [1-101]

    A large pool of adults earning their first driver's license includes 50% low-risk drivers, 30% moderate-risk drivers, and 20% high-risk drivers. Because these drivers have no prior driving record, an insurance company considers each driver to be randomly selected from the pool. This month, the insurance company writes 4 new policies for adults earning their first driver's license. What is the probability that these 4 will contain at least two more high-risk drivers than low-risk drivers?

    (A) 0.006 (B) 0.012 (C) 0.018 (D) 0.049 (E) 0.073

23. [1-102]

    A study of automobile accidents produced the following data:

Model year	Proportion of all vehicles	Probability of involvement in an accident
1997	0.16	0.05
1998	0.18	0.02
1999	0.20	0.03
Other	0.46	0.04

An automobile from one of the model years 1997, 1998, and 1999 was involved in an accident. Determine the probability that the model year of this automobile is 1997.

(A) 0.22 (B) 0.30 (C) 0.33 (D) 0.45 (E) 0.50

24. [1-103]

An insurance company examines its pool of auto insurance customers and gathers the following information:

(i) All customers insure at least one car.
(ii) 70% of the customers insure more than one car.
(iii) 20% of the customers insure a sports car.
(iv) Of those customers who insure more than one car, 15% insure a sports car.

Calculate the probability that a randomly selected customer insures exactly one car and that car is not a sports car.

(A) 0.13 (B) 0.21 (C) 0.24 (D) 0.25 (E) 0.30

25. [1-104]

An auto insurance company has $10,000$ policyholders. Each policyholder is classified as

(i) young or old;
(ii) male or female; and
(iii) married or single.

Of these policyholders, 3000 are young, 4600 are male, and 7000 are married. The policyholders can also be classified as 1320 young males, 3010 married males, and 1400 young married persons. Finally, 600 of the policyholders are young married males. How many of the company's policyholders are young, female, and single?

(A) 280 (B) 423 (C) 486 (D) 880 (E) 896

26. [1-105]

A dental insurance policy covers three procedures: orthodontics, fillings, and extractions. During the life of the policy, the probability that the policyholder needs

- orthodontic work is $1/2$;
- orthodontic work or a filling is $2/3$;

- orthodontic work or an extraction is 3/4,
- a filling and an extraction is 1/8.

The need for orthodontic work is independent of the need for a filling and is independent of the need for an extraction. Calculate the probability that the policyholder will need a filling or an extraction during the life of the policy.

(A) 7/24 (B) 3/8 (C) 2/3 (D) 17/24 (E) 5/6

27. [1-106]

An actuary is studying the prevalence of three health risk factors, denoted by A, B, and C, within a population of women. For each of the three factors, the probability is 0.1 that a woman in the population has only this risk factor (and no others). For any two of the three factors, the probability is 0.12 that she has exactly these two risk factors (but not the other). The probability that a woman has all three risk factors, given that she has A and B, is 1/3. What is the probability that a woman has none of the three risk factors, given that she does not have risk factor A?

(A) 0.280 (B) 0.311 (C) 0.467 (D) 0.484 (E) 0.700

28. [1-107]

An insurance company designates 10% of its customers as high risk and 90% as low risk. The number of claims made by a customer in a calendar year is Poisson distributed with mean θ and is independent of the number of claims made by a customer in the previous calendar year. For high risk customers, $\theta = 0.6$, while for low risk customers $\theta = 0.1$. Calculate the expected number of claims made in calendar year 1998 by a customer who made one claim in calendar year 1997.

(A) 0.15 (B) 0.18 (C) 0.24 (D) 0.30 (E) 0.40