Math 370, Spring 2008
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Practice Test 2

About this test. This is a practice test made up of a random collection of 15 problems from past Course 1/P actuarial exams. Most of the problems have appeared on the Actuarial Problem sets passed out in class, but I have also included some additional problems.

Topics covered. This test covers the topics of Chapters 1–5 in Hogg/Tanis and Actuarial Problem Sets 1–5.

Ordering of the problems. In order to mimick the conditions of the actual exam as closely as possible, the problems are in no particular order. Easy problems are mixed in with hard ones. In fact, I used a program to select the problems and to put them in random order, with no human intervention. If you find the problems hard, it's the luck of the draw!

Suggestions on taking the test. Try to take this test as if it were the real thing. Take it as a closed books, notes, etc., time yourself, and stop after 2 hours. In the actuarial exam you have 3 hours for 30 problems, so 2 is an appropriate time limit for a 20 problem test.

Answers/solutions. Answers and solutions will be posted on the course webpage, `www.math.uiuc.edu/~hildebr/370`.

1. Let X and Y denote the values of two stocks at the end of a five-year period. X is uniformly distributed on the interval $(0, 12)$. Given $X = x$, Y is uniformly distributed on the interval $(0, x)$. Determine $\text{Cov}(X, Y)$ according to this model.

   (A) 0 (B) 4 (C) 6 (D) 12 (E) 24

2. A device contains two circuits. The second circuit is a backup for the first, so the second is used only when the first has failed. The device fails when and only when the second circuit fails. Let X and Y be the times at which the first and second circuits fail, respectively. X and Y have joint probability density function

$$f(x,y) = \begin{cases} 6e^{-x}e^{-2y} & \text{for } 0 < x < y < \infty, \\ 0 & \text{otherwise.} \end{cases}$$

What is the expected time at which the device fails?

(A) 0.33 (B) 0.50 (C) 0.67 (D) 0.83 (E) 1.50

3. An actuary has discovered that policyholders are three times as likely to file two claims as to file four claims. If the number of claims filed has a Poisson distribution, what is the variance of the number of claims filed?

   (A) $1/\sqrt{3}$ (B) 1 (C) $\sqrt{2}$ (D) 2 (E) 4

4. An insurance company examines its pool of auto insurance customers and gathers the following information:

   (i) All customers insure at least one car.

   (ii) 70% of the customers insure more than one car.

   (iii) 20% of the customers insure a sports car.

   (iv) Of those customers who insure more than one car, 15% insure a sports car.

   Calculate the probability that a randomly selected customer insures exactly one car and that car is not a sports car.

   (A) 0.13 (B) 0.21 (C) 0.24 (D) 0.25 (E) 0.30

5. A device that continuously measures and records seismic activity is placed in a remote region. The time, T, to failure of this device is exponentially distributed with mean 3 years. Since the device will not be monitored during its first two years of service, the time to discovery of its failure is $X = \max(T, 2)$. Determine $E(X)$.

   (A) $2 + \frac{1}{3}e^{-6}$

   (B) $2 - 2e^{-2/3} + 5e^{-4/3}$

   (C) 3

   (D) $2 + 3e^{-2/3}$

   (E) 5

6. A car dealership sells 0, 1, or 2 luxury cars on any day. When selling a car, the dealer also tries to persuade the customer to buy an extended warranty for the car. Let X denote the number of luxury cars sold in a given day, and let Y denote the number of extended warranties sold, and suppose that

$$P(X = x, Y = y) = \begin{cases} 1/6 & \text{for } (x, y) = (0, 0), \\ 1/12 & \text{for } (x, y) = (1, 0), \\ 1/6 & \text{for } (x, y) = (1, 1), \\ 1/12 & \text{for } (x, y) = (2, 0), \\ 1/3 & \text{for } (x, y) = (2, 1), \\ 1/6 & \text{for } (x, y) = (2, 2). \end{cases}$$

What is the variance of X?

(A) 0.47 (B) 0.58 (C) 0.83 (D) 1.42 (E) 2.58

7. A company prices its hurricane insurance using the following assumptions:

   (i) In any calendar year, there can be at most one hurricane.
   (ii) In any calendar year, the probability of a hurricane is 0.05.
   (iii) The number of hurricanes in any calendar year is independent of the number of hurricanes in any other calendar year.

   Using the company's assumptions, calculate the probability that there are fewer than 3 hurricanes in a 20-year period.

   (A) 0.06 (B) 0.19 (C) 0.38 (D) 0.62 (E) 0.92

8. The loss due to a fire in a commercial building is modeled by a random variable X with density function
$$f(x) = \begin{cases} 0.005(20 - x) & \text{for } 0 < x < 20, \\ 0 & \text{otherwise.} \end{cases}$$
Given that a fire loss exceeds 8, what is the probability that it exceeds 16?

(A) 1/25 (B) 1/9 (C) 1/8 (D) 1/3 (E) 3/7

9. A large pool of adults earning their first driver's license includes 50% low-risk drivers, 30% moderate-risk drivers, and 20% high-risk drivers. Because these drivers have no prior driving record, an insurance company considers each driver to be randomly selected from the pool. This month, the insurance company writes 4 new policies for adults earning their first driver's license. What is the probability that these 4 will contain at least two more high-risk drivers than low-risk drivers?

   (A) 0.006 (B) 0.012 (C) 0.018 (D) 0.049 (E) 0.073

10. An insurance company determines that N, the number of claims received in a week, is a random variable with $P(N = n) = 2^{-n-1}$, where $n \geq 0$. The company also determines that the number of claims received in a given week is independent of the number of claims received in any other week. Determine the probability that exactly seven claims will be received during a given two-week period.

    (A) $\frac{1}{256}$ (B) $\frac{1}{128}$ (C) $\frac{7}{512}$ (D) $\frac{1}{64}$ (E) $\frac{1}{32}$

11. A device contains two components. The device fails if either component fails. The joint density function of the lifetimes of the components, measured in hours, is $f(s,t)$, where $0 < s < 1$ and $0 < t < 1$. What is the probability that the device fails during the first half hour of operation?

    (A) $\int_0^{0.5}\int_0^{0.5} f(s,t)dsdt$

    (B) $\int_0^{1}\int_0^{0.5} f(s,t)dsdt$

    (C) $\int_{0.5}^{1}\int_{0.5}^{1} f(s,t)dsdt$

    (D) $\int_0^{0.5}\int_0^{1} f(s,t)dsdt + \int_0^{1}\int_0^{0.5} f(s,t)dsdt$

    (E) $\int_0^{0.5}\int_{0.5}^{1} f(s,t)dsdt + \int_0^{1}\int_0^{0.5} f(s,t)dsdt$

12. The time to failure of a component in an electronic device has an exponential distribution with a median of four hours. Calculate the probability that the component will work without failing for at least five hours.

(A) 0.07 (B) 0.29 (C) 0.38 (D) 0.42 (E) 0.57

13. A health study tracked a group of persons for five years. At the beginning of the study, 20% were classified as heavy smokers, 30% as light smokers, and 50% as nonsmokers. Results of the study showed that light smokers were twice as likely as nonsmokers to die during the five-year study, but only half as likely as heavy smokers. A randomly selected participant from the study died over the five-year period. Calculate the probability that the participant was a heavy smoker.

    (A) 0.20 (B) 0.25 (C) 0.35 (D) 0.42 (E) 0.57

14. The stock prices of two companies at the end of any given year are modeled with random variables X and Y that follow a distribution with joint density function

$$f(x,y) = \begin{cases} 2x & \text{for } 0 < x < 1,\ x < y < x+1, \\ 0 & \text{otherwise.} \end{cases}$$

What is the conditional variance of Y given that $X = x$?

(A) $1/12$ (B) $7/6$ (C) $x + 1/2$ (D) $x^2 - 1/6$ (E) $x^2 + x + 1/3$

15. An insurance company sells an auto insurance policy that covers losses incurred by a policyholder, subject to a deductible of 100. Losses incurred follow an exponential distribution with mean 300. What is the 95th percentile of actual losses that exceed the deductible?

    (A) 600 (B) 700 (C) 800 (D) 900 (E) 1000

16. An insurance company insures a large number of drivers. Let X be the random variable representing the company's losses under collision insurance, and let Y represent the company's losses under liability insurance. X and Y have joint density function

$$f(x, y) = \begin{cases} \frac{1}{4}(2x + 2 - y) & \text{for } 0 < x < 1 \text{ and } 0 < y < 2, \\ 0 & \text{otherwise.} \end{cases}$$

What is the probability that the total loss is at least 1?

(A) 0.33 (B) 0.38 (C) 0.41 (D) 0.71 (E) 0.75

17. An insurance company sells a one-year automobile policy with a deductible of 2. The probability that the insured will incur a loss is 0.05. If there is a loss, the probability of a loss of amount N is K/N, for $N = 1, \ldots, 5$ and K a constant. These are the only possible loss amounts and no more than one loss can occur. Determine the net premium for this policy.

    (A) 0.031 (B) 0.066 (C) 0.072 (D) 0.110 (E) 0.150

18. An insurance policy pays a total medical benefit consisting of a part paid to the surgeon, X, and a part paid to the hospital, Y, so that the total benefit is $X + Y$. It is known that $\mathrm{Var}(X) = 5,000$, $\mathrm{Var}(Y) = 10,000$, and $\mathrm{Var}(X + Y) = 17,000$. Due to increasing medical costs, the company that issues the policy decides to increase X by a flat amount of 100 per claim and to increase Y by 10% per claim. Calculate the variance of the total benefit after these revisions have been made.

    (A) 18,200 (B) 18,800 (C) 19,300 (D) 19,520 (E) 20,670

19. Let T_1 be the time between a car accident and reporting a claim to the insurance company. Let T_2 be the time between the report of the claim and payment of the claim. The joint density function of T_1 and T_2, $f(t_1, t_2)$, is constant over the region $0 < t_1 < 6, 0 < t_2 < 6, 0 < t_1 + t_2 < 10$, and zero otherwise. Determine $E(T_1 + T_2)$, the expected time between a car accident and payment of the claim.

    (A) 4.9 (B) 5.0 (C) 5.7 (D) 6.0 (E) 6.7

20. A company agrees to accept the highest of four sealed bids on a property. The four bids are regarded as four independent random variables with common cumulative distribution function

$$F(x) = \frac{1}{2}(1 + \sin \pi x) \quad \text{for } 3/2 \leq x \leq 5/2.$$

Which of the following represents the expected value of the accepted bid?

(A) $\pi \int_{3/2}^{5/2} x \cos \pi x dx$

(B) $\frac{1}{16} \int_{3/2}^{5/2} (1 + \sin \pi x)^4 dx$

(C) $\frac{1}{16} \int_{3/2}^{5/2} x(1 + \sin \pi x)^4 dx$

(D) $\frac{\pi}{4} \int_{3/2}^{5/2} (\cos \pi x)(1 + \sin \pi x)^3 dx$

(E) $\frac{\pi}{4} \int_{3/2}^{5/2} x(\cos \pi x)(1 + \sin \pi x)^3 dx$