Math 370, Spring 2008
Prof. A.J. Hildebrand

Practice Test 1

March 31, 2008

About this test. This is a practice test made up of a random collection of 15 problems from past Course 1/P actuarial exams. Most of the problems have appeared on the Actuarial Problem sets passed out in class.

Topics covered. This test covers the topics of Chapters 1–3 in Hogg/Tanis and Actuarial Problem Sets 1–3: General probability, discrete distributions, and continuous distributions.

Ordering of the problems. In order to mimick the conditions of the actual exam as closely as possible, the problems are in no particular order. Easy problems are mixed in with hard ones. In fact, I used a program to select the problems and to put them in random order, with no human intervention. If you find the problems hard, it's the luck of the draw!

Suggestions on taking the test. Try to take this test as if it were the real thing. Take it as a closed books, notes, etc., time yourself, and stop after 1:30 hours. In the actuarial exam you have 3 hours for 30 problems, so 1:30 is an appropriate time limit for a 15 problem test.

Answers/solutions. Answers and solutions will be posted on the course webpage, `www.math.uiuc.edu/~hildebr/370`.

1. An actuary is reviewing a study she performed on the size of the claims made ten years ago under homeowners insurance policies. In her study, she concluded that the size of claims followed an exponential distribution and that the probability that a claim would be less than \$1,000 was 0.250. The actuary feels that the conclusions she reached in her study are still valid today with one exception: every claim made today would be twice the size of a similar claim made ten years ago as a result of inflation. Calculate the probability that the size of a claim made today is less than \$1,000.

2. A study is being conducted in which the health of two independent groups of ten policyholders is being monitored over a one-year period of time. Individual participants in the study drop out before the end of the study with probability 0.2 (independently of the other participants). What is the probability that at least 9 participants complete the study in one of the two groups, but not in both groups?

   (A) 0.096 (B) 0.192 (C) 0.235 (D) 0.376 (E) 0.469

3. An actuary studied the likelihood that different types of drivers would be involved in at least one collision during any one-year period. The results of the study are presented below.

Type of driver	Percentage of all drivers	Probability of at least one collision
Teen	8%	0.15
Young adult	16%	0.08
Midlife	45%	0.04
Senior	31%	0.05

Given that a driver has been involved in at least one collision in the past year, what is the probability that the driver is a young adult driver?

(A) 0.06 (B) 0.16 (C) 0.19 (D) 0.22 (E) 0.25

4. An insurance company determines that N, the number of claims received in a week, is a random variable with $P(N = n) = 2^{-n-1}$, where $n \geq 0$. The company also determines that the number of claims received in a given week is independent of the number of claims received in any other week. Determine the probability that exactly seven claims will be received during a given two-week period.

   (A) $\frac{1}{256}$ (B) $\frac{1}{128}$ (C) $\frac{7}{512}$ (D) $\frac{1}{64}$ (E) $\frac{1}{32}$

5. An insurance policy pays an individual 100 per day for up to 3 days of hospitalization and 25 per day for each day of hospitalization thereafter. The number of days of hospitalization, X, is a discrete random variable with probability function

$$P(X = k) = \begin{cases} \frac{6-k}{15} & \text{for } k = 1, 2, 3, 4, 5, \\ 0 & \text{otherwise.} \end{cases}$$

Calculate the expected payment for hospitalization under this policy.

(A) 85 (B) 163 (C) 168 (D) 213 (E) 255

6. An actuary has discovered that policyholders are three times as likely to file two claims as to file four claims. If the number of claims filed has a Poisson distribution, what is the variance of the number of claims filed?

(A) $1/\sqrt{3}$ (B) 1 (C) $\sqrt{2}$ (D) 2 (E) 4

7. A device that continuously measures and records seismic activity is placed in a remote region. The time, T, to failure of this device is exponentially distributed with mean 3 years. Since the device will not be monitored during its first two years of service, the time to discovery of its failure is $X = \max(T, 2)$. Determine $E(X)$.

   (A) $2 + \frac{1}{3}e^{-6}$

   (B) $2 - 2e^{-2/3} + 5e^{-4/3}$

   (C) 3

   (D) $2 + 3e^{-2/3}$

   (E) 5

8. A tour operator has a bus that can accommodate 20 tourists. The operator knows that tourists may not show up, so he sells 21 tickets. The probability that an individual tourist will not show up is 0.02, independent of all other tourists. Each ticket costs 50, and is non-refundable if a tourist fails to show up. If a tourist shows up and a seat is not available, the tour operator has to pay 100 (ticket cost + 50 penalty) to the tourist. What is the expected revenue of the tour operator?

   (A) 935 (B) 950 (C) 967 (D) 976 (E) 985

9. Under a group insurance policy, an insurer agrees to pay 100% of the medical bills incurred during the year by employees of a small company, up to a maximum total of one million dollars. The total amount of bills incurred, X, has probability density function

$$f(x) = \begin{cases} \frac{x(4-x)}{9} & \text{for } 0 < x < 3, \\ 0 & \text{otherwise,} \end{cases}$$

where x is measured in millions. Calculate the total amount, in millions of dollars, the insurer would expect to pay under this policy.

(A) 0.120 (B) 0.301 (C) 0.935 (D) 2.338 (E) 3.495

10. The loss amount, X, for a medical insurance policy has cumulative distribution function

$$F(x) = \begin{cases} 0, & x < 0, \\ \frac{1}{9}\left(2x^2 - \frac{x^3}{3}\right), & 0 \le x \le 3, \\ 1, & x > 3. \end{cases}$$

Calculate the mode of the distribution.

(A) $\frac{2}{3}$ (B) 1 (C) $\frac{3}{2}$ (D) 2 (E) 3

11. Let X be a continuous random variable with density function

$$f(x) = \begin{cases} \frac{|x|}{10} & \text{for } -2 \leq x \leq 4, \\ 0 & \text{otherwise.} \end{cases}$$

Calculate the expected value of X.

(A) $\frac{1}{5}$ (B) $\frac{3}{5}$ (C) 1 (D) $\frac{28}{15}$ (E) $\frac{12}{5}$

12. An insurance policy pays for a random loss X subject to a deductible of C, where $0 < C < 1$. The loss amount is modeled as a continuous random variable with density function

$$f(x) = \begin{cases} 2x & \text{for } 0 < x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

Given a random loss X, the probability that the insurance payment is less than 0.5 is equal to 0.64. Calculate C.

(A) 0.1 (B) 0.3 (C) 0.4 (D) 0.6 (E) 0.8

13. An insurance policy is written to cover a loss, X, where X has a uniform distribution on $[0, 1000]$. At what level must a deductible be set in order for the expected payment to be 25% of what it would be with no deductible?

(A) 250 (B) 375 (C) 500 (D) 625 (E) 750

14. An insurance policyholder can submit up to 5 claims. The probability that the policyholder submits exactly n claims is p_n, for $n = 0, 1, 2, 3, 4, 5$. The following information is known:

    (i) The difference between p_n and p_{n+1} is constant for $n = 0, 1, 2, 3, 4$.

    (ii) Exactly 40% of the policyholders submit fewer than two claims.

    What is the probability that a policholder submits more than three claims?

    (A) 0.06 (B) 0.19 (C) 0.26 (D) 0.34 (E) 0.45

15. A company offers a health insurance plan, a life insurance plan, and an investment insurance plan. An employee can have 0, 1, or 2 plans, but cannot have both life insurance and investment plans. You are given the following information:

    (i) 450 employees have at least one plan.

    (ii) 330 employees have only one plan.

    (iii) 320 employees have the health insurance plan.

    (iv) 45 employees have only the life insurance plan.

    (v) There are 20 more employees that have both health and life plans than those that have both health and investment plans.

    How many people have the investment plan?

    (A) 85 (B) 120 (C) 135 (D) 155 (E) 180