Practice Problems on Integrals

1. Evaluate the following integrals:

   (a) $\int_0^1 (x^3 + 2x^5 + 3x^{10})dx$
   (b) $\int_0^\infty (1+x)^{-5}dx$
   (c) $\int_0^\infty x(1+x)^{-5}dx$
   (d) $\int_1^\infty e^{-3x}dx$
   (e) $\int_1^\infty xe^{-3x}dx$
   (f) $\int_{-\infty}^\infty |x|e^{-x^2/2}dx$

2. Given that X has density (p.d.f.)

$$f(x) = \begin{cases} 1 - |x| & \text{for } -1 < x < 1, \\ 0 & \text{otherwise,} \end{cases}$$

evaluate:

(a) $P(X \geq 1/2)$
(b) $P(X \geq -1/2)$
(c) $E(X)$
(d) $E(X^2)$
(e) $F(x)$ (the c.d.f.)

3. Let X be exponentially distributed with mean 2. Determine:

   (a) $P(X \geq 5)$.
   (b) $P(2 \leq X \leq 5)$.
   (c) $P(2 < X < 5)$.
   (d) $P(X \geq 5 | X \geq 2)$.
   (e) $P(X \leq 5 | X \geq 2)$.

4. Suppose X has exponential distribution with median 3. Determine:

   (a) $E(X)$.

   (b) The 75-th percentile of the distribution of X.

5. Let X be exponentially distributed with mean 2, and let Y be defined by

$$Y = \begin{cases} 0 & \text{if } X \le 1, \\ X - 1 & \text{if } X > 1. \end{cases}$$

Find $E(Y)$.

6. Let X be exponentially distributed with mean 2, and let

$$Y = \begin{cases} X & \text{if } X \le 5, \\ 5 & \text{if } X > 5. \end{cases}$$

Find $E(Y)$.

7. Let X be exponentially distributed with mean 2, and let Y be defined by

$$Y = \begin{cases} X & \text{if } X \le 1, \\ (1/2)(X+1) & \text{if } X > 1. \end{cases}$$

Find $E(Y)$.

8. Let X be exponentially distributed with mean 3, and let $Y = \max(X, 2)$. Find $E(Y)$.

9. Assume the amount of damage, X, in an auto accident is exponentially distributed with mean 2. (All figures are thousands of dollars.)

   (a) Suppose first the insurance company covers the actual amount of the loss, up to a maximum of 5. What is the average payoff?

   (b) Suppose now the insurance company covers the full amount of the loss minus a deductible of 1. What is the average payoff?

   (c) Suppose the insurance company covers the full amount of the loss up to 1, and 50% of any loss in excess of 1. What is the average payoff?